High-Frequency Gravitational Wave Overview
STAIF2008 Plenary Session

1:45 pm
Revised May 27, 2008
What are High-Frequency Gravitational Waves (HFGWs) ?

High Frequency Gravity Waves (HFGWs) are gravitational waves (which differ from hydrodynamic gravity waves) that have frequencies of 100 kHz to 100 MHz (Hawking and Israel 1979). As their frequency increases, very high-frequency gravitational waves (VHFGWs) and ultra high-frequency gravitational waves (UHFGWs) are generated. These have frequencies of 100 MHz to 100 GHz and 100+ GHz, respectively. The generic term HFGWs describes all three of these bands. These waves move through the “fabric” of space similar to the way an ocean wave moves through the water. This “fabric,” as Einstein describes it, is called the “space-time continuum” and is four-dimensional. It contains the usual three dimensions of space: for example, (1) east-west (2) north-south) (3) up down, but also includes the dimension of (4) time.
Gravitational Waves Analogies
History of High-Frequency Gravitational Waves

- Poincaré, first mention of GWs, 1905
- Robert L. Forward, lecture presented at the *Lockheed Astrodynamics Research Center* in Bel-Air, California, USA, first mention of HFGWs, 1961
- M. E. Gertsenshtein, 1962
- L. Halpern and B. Laurent, relic HFGWs from Big Bang & GASER, 1964
- Richard A. Isaacson, 1968
- Leonid Grishchuk and M. V. Sazhin HFGW emission, 1974
- V. B. Braginsky and Valentin N. Rudenko, detection and laboratory HFGW generation, 1978
- 1979 Steven W. Hawking and W. Israel definition for HFGWs as having frequencies in excess of 100 kHz.
- G. Veneziano, M. Gasperini and M. Giovannini relic HFGWs from Big Bang, 1990
- Fangyu Li and R. Baker, Proposed HFGW detector, 1999 to date
- First HFGW patent, R. Baker, 1999
- M. Cruise and R. Ingley built HFGW detector, *Birmingham U.*, 2000 to date
HFGW Literature

- *Amaldi and Elba Gravitational Wave Conferences*: M. Cruise, R. Inglcy, Philippe Bernard, Gianluca Gemme, R. Parodi, and E. Picasso
- For more complete list of HFGW references please enter site [www.GravWave.com](http://www.GravWave.com), LITERATURE SURVEY.
2nd HFGW International Workshop
Institute for Advanced Studies at Austin (IASA), Texas, September 17-20, 2007

- Biao Li, Jie Zhen, Leonid Grishchuk, Zhenyun Fang, Kit Green, Fangyu Li, Bob Baker, Gary Stephenson, and Hal Puthoff
Chinese Faculty and Students involved in HFGW Research,
Chongqing University, China April 12, 2008
What is there to detect?

“High-Frequency Relic Gravitational Waves and Their Detection”

L. P. Grishchuk

Cardiff University, United Kingdom
and Moscow State University, Russia
E-mail: grishchuk@astro.cf.ac.uk
HFGW2 Workshop, Austin, Texas, 19 September, 2007
www.earthtech.org/hfgw2/
Where HFGWs come from:
Grishchuk found the Energy density of relic gravitational waves

\[ \Omega_{gw}(\nu) = \frac{\pi^2}{3} h^2(\nu) \left( \frac{\nu}{\nu_H} \right)^2 \]
Sensitivity \( h \) as function of \( v_{GW} \) and Hubble parameter \( n \) according to Grishchuk
Relic gravitational waves will allow us to make direct inferences about the early universe Hubble parameter and scale factor ("birth" of the Universe and its early dynamical evolution). According to Grishchuk:

- Energy density requires that the GW frequency be on the order of $10^{10}$ Hz (10 GHz)
- Sensitivity required for that frequency on the order of $10^{-30} \, \delta m/m$
Recent Analyses

- Primordial gravitational waves are a robust prediction of inflation as they are produced by the same mechanism that generated the primordial density fluctuations observed in the CMB (Cosmic Microwave Background) and LSS (Large Scale Structure) data.

- Recently (March 2008), a team of theorists from Paris Observatory together with Claudio Destri from INFN/University of Milano-Bicocca performed a new analysis of all available CMB and LSS data including the WMAP and Sloan data and favor an inflation model where there must exist primordial gravitational waves of high frequency.
Why LIGO, Virgo, GEO600, et al. can’t detect HFGWs

The advertised Laser Interferometer Gravitational Observatory (LIGO) sensitivity (see, for example, "Gravitational Waves and the Effort to Detect them," in American Scientist, Volume 42, July-August, 2004, p. 356) is 40Hz to 2000Hz. Here's the problem with higher frequencies: One has to "observe" the interference pattern between the LIGO legs caused by the passage of a gravitational wave. From the referenced article: "At higher frequencies, the quantum nature of the laser beam (made of discrete photons, albeit a large number of them) limits the precision of the measurement. Increased laser power would reduce the problem of quantum noise, but ultimately the LIGO (and other) interferometers (such as Virgo, GEO600, Advanced LIGO and the proposed Laser Interferometer Space Antenna or LISA) are not suited to measuring gravitational waves that stretch or shrink the arms much more rapidly than the time a photon typically remains in the optical cavity, which is roughly a millisecond for these interferometers (thus a one kilocycle frequency)." That's why one must turn to the Birmingham University, England; the INFN Genoa, Italy; Japanese 100 MHz and the Chongqing University, China HFGW Detectors.
Where HFGWs come from:
Laboratory Sources of HFGWs

- GASER: L. Halpern, B. Laurent and Giorgio Fontana
- Dual LASER Generator: R. Baker, Fangyu Li and Ruxin Li
- Nuclear generation: Fontana and Baker
- Piezoelectric Crystals (Dehnen and Romero-Borja) and Film Bulk Acoustic Resonators or FBARs (Baker, Woods, Stephenson and Li) and IR Excited Masses in a Ring Component of a Cylindrical HFGW Generator (Woods and Baker)
Depiction of the Magnetron-FBAR HFGW Generator Design

Thousands of Magnetrons
Similar to those Found in Microwave Ovens

2.45 GHz Microwaves

Millions of FBARs
Similar to those Found in Cell Phones

4.9 GHz HFGWs Created by the Jorla in the Piezoelectric Crystal in the FBARs

GRAVWAVE ® LLC.
Woods Concept of a HFGW Generation Ring

The easiest configuration to analyze requires that two counter-propagating traveling waves be excited inside this waveguide, thus producing a standing optical wave inside the guide.

Regions of the guide separated by $\lambda/2$ will oscillate in antiphase but the resultant HFGW will be in phase, since the produced GW is at doubled frequency.

Therefore, this configuration is equivalent to the ring of discrete acoustic resonators (small masses) proposed previously by Woods and Baker (2005) for terrestrial HFGW production.

Because the active material vibrates in phase and in opposite pairs and has circular symmetry, all the generated GW will combine in phase at the center of the torus.
Woods Concept

Individual Ring

IR monomode waveguide formed by active material

Mirrors

Position of focused GW generated, normal to page

High power IR source

WOODS & BAKER HFGW GENERATOR

- ELECTRONIC OR ATOMIC HFGW GENERATOR – CIRCULAR GEOMETRY IN A STACK OR BARRELL
Birmingham HFGW Detector

- Concept Developed by Mike Cruise at Birmingham University, England and

Birmingham (Polarization) HFGW Detector
(Differential Polarization Angle of $10^{-40}$ Radians may Cause Measurable Femtosecond Time Difference)
Sensitivity

• The best sensitivity of the Cruise & Ingley HFGW detector is obtained at those frequencies where the noise floor is lowest. This puts the optimum strain sensitivity of the detectors around $2 \times 10^{-14}/\sqrt{\text{Hz}}$ and $6 \times 10^{-14}/\sqrt{\text{Hz}} \, \delta\text{m/m}$.

• The strain sensitivity of the cross correlated detectors is to first order $\sqrt{Q}$ times smaller than the angular amplitude sensitivity. This places the final sensitivity of the instruments at a dimensionless strain at $5 \times 10^{-14}/\sqrt{\text{Hz}} \, \delta\text{m/m}$.

• As yet the relic or HFRGW detection requirements for the Cruise & Ingley or Birmingham University detectors are not yet met.
• In the scheme suggested by Bernard et al. HFGWs are detected by coupling two identical high frequency cavities. Each resonant mode of the individual cavity is then split in two modes of the coupled resonator with different spatial field distribution and a passing HFGW is sensed by measuring a slight change in resonance frequency of the two cavities. The resonance frequencies of the cavities are about ten times the HFGW frequencies and they are about a wavelength in dimension so for 10 GHz HFRGWs the size of the detector is millimeters and difficult to build.

Niobium spherical cavities (variable coupling)
Sensitivity

• The theoretical ultimate sensitivity for GHz frequencies as a result of the optimization: $h_{\text{min}} \approx 2.2 \times 10^{-22} \text{ Hz}^{-1/2} \delta m/m$.

• The HFRGW detection sensitivity requirements for this INFN Genoa detector are not yet met.
Mike Cruise 17 May 2008
Elba Conference Slide

• Kawamura Japanese Detector

Development of 100MHz GW detectors at National Astronomical Observatory of Japan

- Arm length: 75 cm
- Aimed GW frequency: 100 MHz
- Finesse: 100

Two synchronous recycling interferometers were built!

Synchronous recycling Interferometer (Concept: Drever 1983)
Future Challenges

- Japanese Detector
  - Already achieving $h=10^{-16}$ per root Hz
  - Increase finesse to $1.5 \times 10^5$
  - Ultimate limit will be photon noise
  - This will limit higher frequency detections
  - Current system can be developed to greater sensitivity at $v=100$ MHz, but this may be too low for interesting Stochastic Models
  - Small and compact - good for correlation work
Geometry is key:
X & -X axes = Detectors
Y axis = Magnetic Field
Z axis = HFRGW
-Z axis = Gaussian Beam
Conclusions regarding Chinese HFGW detector

• Early studies indicate that for $h = 10^{-30}$ to $10^{-32}$ $\delta$m/m amplitude HFRGW signals, the foregoing detection scheme is theoretically sufficient to detect them – and specifically designed for that purpose.

• The active approach, using synchro-resonance and fractal membranes, is required to achieve the needed sensitivity for these weak signals ($<<10^{-25}$ $\delta$m/m).

Next Steps:
• Develop additional detail for every stage of the experimental design as in the Chinese Statement of Work.
• Use the more detailed design data to prepare a more detailed sensitivity analysis, (“Phase A Study.”)
• Based on the results of the Phase A Study, determine sources of funding for a detailed experimental design and risk analysis, (“Phase B Study.”) These are the steps the Chinese are taking.
Practical Applications of HFGW Generators and Detectors

• **Telecommunications:** Thomas Prince (Chief Scientist, NASA/JPL and Professor of Physics at Caltech) commented (2002): “Of the applications (of HFGWs), communication would seem to be the most important. Gravitational waves have a very low cross section for absorption by normal matter and therefore high-frequency waves could, in principle, carry significant information content with effectively no absorption unlike any electromagnetic waves.” Potential for broad-band communication to deeply submerged submarines. Harper and Stephenson find cost savings in communications message search-space and frequency-reference improvement and in phase-noise reduction of up to 150 billion dollars over ten years.

• **Propulsion:** “Since it has definite energy, the gravitational wave is itself the source of some additional gravitational field... its field is a second-order effect ... But in the case of high-frequency gravitational waves the effect is significantly strengthened...” Landau & Lifshitz. Thus it is possible to change the gravitational field near an object remotely by means of HFGWs and move or perturb it. A totally new propulsion means
Practical Applications Continued

- **Optics:** According to a rather controversial theory a superconductor exhibits a large index of refraction for GWs (Li and Torr 1992). Thus optical devices, such as astronomical telescopes (both refracting and reflecting (Baker 2000 and 2004)), communication-link concentrators (Woods 2005), and variable-focus HFGW optical systems (Woods 2006 and 2007) can be designed and utilized in practice.

- **Surveillance:** Although very speculative there is the potential for through-earth, or through-water “X-rays” utilizing the extreme sensitivity of HFGW generation-detection systems to polarization angle changes (possibly sensitive to even less than $10^{-40}$ radians or one billion, billion, billion, billionth of a degree change) can be utilized in order to observe subterranean structures, geological formations (such as oil deposits), create a transparent ocean, view three-dimensional building interiors, buried devices, hidden missiles, weapons of mass destruction, achieve remote acoustical surveillance or eavesdropping, etc.
Nuclear Fusion: If there is an ultra-high intensity HFGW flux impinging on a nucleus, then it is possible to initiate nuclear fusion at a remote location – mass disruption. Also it may be possible to create radioactive waste-free nuclear reactions and energy creation (Fontana, G. and Baker, R. M L, Jr. 2007). This paper mainly explored the possibility suggested by Fontana that an intense beam of HFGWs might be employed to change the relativistic mass of elementary particles and clusters of elementary particles in combination with space-time compression. The paper focused on the application of the predicted phenomenon for nuclear fusion and energy production, mimicking the very real muonic fusion.
Conclusions Primarily Based Upon the Research Results of Others

• HFGW research is based upon almost five decades of research resulting in hundreds peer-reviewed scientific papers and the creation of laboratory hardware.

• New technology will allow for the generation of HFGWs in the laboratory.

• New technology will also allow for the design of ultra-high sensitivity HFGW detectors capable of detecting relic HFGWs and those generated in the laboratory.

• Practical applications of HFGW research may include:
  telecommunications
  propulsion
  optics
  surveillance
  remotely activated nuclear reactions

For more details please visit: www.GravWave.com
GENERATION OF GRAVITATIONAL WAVES

Excerpts from:


Indirect evidence obtained by J. H. Taylor and R. A. Hulse (1970s) concerning their observations of a contracting binary star pair or pulsar PSR 1913+16, which perfectly matched Einstein’s GW theory, garnered them the 1993 Nobel Prize and the skepticism concerning GW evaporated,


GW Radiation Based on General Relativity (GR) as discussed in Landau and Lifschitz (1975) p. 356

GRavitational waves

§ 110

PROBLEMS

1. Two bodies, attracting each other according to Newton’s law, move in circular orbits (around their common center of inertia). Determine the average (over a rotation period) of the intensity of radiation of gravitational waves and its distribution in polarization and direction.

Solution: Choosing the coordinate origin at the center of inertia, we have for the radius vectors of the two bodies:

\[ r_1 = \frac{m_1}{m_1 - m_2} r, \quad r_2 = \frac{m_2}{m_1 - m_2} r - r_1 - r_2. \]

The components of the tensor \( D_{ij} \) are (if the xy plane coincides with the plane of motion):

\[
D_{xx} = \mu r^2 (3 \cos^2 \varphi - 1), \quad D_{yy} = \mu r^2 (3 \sin^2 \varphi - 1),
\]

\[
D_{xy} = 3 \mu r^2 \cos \psi \sin \psi, \quad D_{zz} = -\mu r^2,
\]

where \( \mu = m_1 m_2 (m_1 + m_2) \), \( \varphi \) is the polar angle of the vector \( r \) in the xy plane. For circular motion \( r = \text{const} \), and \( \psi = r - \sqrt{k} (m_1 + m_2) = \omega t \).

We assign the direction of \( n \) by the polar angle \( \theta \) and azimuth \( \phi \), with the polar axis \( z \) perpendicular to the plane of the motion. Let us consider the two polarizations for which: (1) \( e_{\phi} = 1 / \sqrt{2} \); (2) \( e_{\theta} = -e_{\phi} = 1 / \sqrt{2} \). Projecting the tensor \( D_{ij} \) on the directions of the spherical unit vectors \( e_\phi \) and \( e_\theta \), calculating with formula (110.13) and averaging over the time, we find the result for these two cases and for the sum \( I = I_\phi + I_\theta \):

\[
\frac{dI}{d\omega} = \frac{k \mu^2 r^4 \omega^2}{2 \pi c^5}, \quad \frac{dI}{d\omega} = \frac{k \mu^2 r^4 \omega^2}{2 \pi c^5} (1 - \cos^2 \theta)^2.
\]

\[
\frac{dI}{d\omega} = \frac{k \mu^2 r^4 \omega^2}{2 \pi c^5} (1 + 6 \cos^2 \theta + \cos^4 \theta),
\]
GW GENERATED BY ORBITING MASSES AND BY TWO JERKING MASSES
The Radiation Pattern Obtained from the Landau & Lifshitz Equations is Peanut Shaped
A Linear Array of GW “Sources”

For example a stack of Orbit Planes
We utilize the result of Romero and Dehnen (1981) and Dehnen and Romero-Borja (2003) for an increase in HFGW flux (in a linear array of $N$ in-phase radiation elements such as their crystal oscillators or vibrators) called “Superradiance” proportional to $N^2$. [Scully and Svidzinsky. (2009), Science 325, pp.1510-1511] as shown in the next slide,

This result is employed to compute the half-power-point angle, idealized radiation cap area and the HFGW flux/power-of-a-single-radiation-element at a distance of several wavelengths away, for example one meter from the end of a linear array in Wm$^{-2}$ as a function of $N$.

The notional picture shown of an idealized needle-like radiation beam is in the far field. It is described at a distance far enough from the generator that it is beyond the conventional diffraction limit of a beam’s radiation-pattern cap area.
Figure 3: Angular distribution for the gravitational radiation of the vibrator-rod (formula (2.21) with \( N = 10^4, \Omega = 10^9 \text{ sec}^{-1} \) and \( a = 0.5 \text{ cm} \); the values for \( \log T_{gw} \) correspond alone to the angular part of \( T_{gw}^{\theta \phi} \):  

a) \( \phi = \pi/2, 3\pi/2; \theta \) runs  
b) \( \phi = \pi/2, 3\pi/2; \theta \) runs. Evidently there exist two well distinguished directions of
The GW Beam Produced by Superradiance

N = 20
N = 19
N = 18
N = 17
N = 16
N = 15
N = 14
N = 13
N = 12
N = 11
N = 10
N = 9
N = 8
N = 7
N = 6
N = 5
N = 4
N = 3
N = 2
N = 1
A single radiation element pair:
3 DERIVATION FROM THE SPINNING-ROD QUADRUPOLE FORMULATION

Principal moment of inertia, \( I \), of a three-dimensional tensor of the system and . . . can be approximated by

\[
-\frac{dE}{dt} = -\frac{G}{5c^5} \left( \frac{d^3I}{dt^3} \right)^2 = -5.5 \times 10^{-54} \left( \frac{d^3I}{dt^3} \right)^2,
\]

or from Eq. (110.16) of L. D. Landau and E. M. Lifshitz (1975):

\[
P = -\frac{dE}{dt} = \kappa \left( \frac{G}{15r^5} \right) \left( \frac{d^3D_{tc}}{dt^3} \right)^2 W
\]

or

\[
P = 1.76 \times 10^{-52} \left( \frac{d^3I}{dt^3} \right)^2 W.
\]

This is Einstein’s quadrupole equation phrased in a different fashion. In Eq. (1), \( G \), the universal gravitational constant = 6.67423 \times 10^{-11} \ m^3/\text{kg} \cdot \text{s}^2 \), \( c \) is the speed of light = 2.99792 \times 10^8 \ m/s \), and the units in Eq. (2) are in the MKS system. In order to introduce the jerk concept concretely, let us consider the hypothetical example of a dumbbell that need not be uniformly rotating or, in fact, not rotating at all; but subjected to an impulsive force. In this case, for the dumbbell-shaped collection of masses,

\[
I = \delta m \ r^2 \quad \text{kg} \cdot \text{m}^2
\]

where \( \delta m \) = mass at either end of the dumbbell, \( kg \), and \( r \) = the distance from a pivot out to \( \delta m \), \( m \), (or more exactly, the radius of gyration of the dumbbell). Thus

\[
\frac{d^2I}{dt^2} = \delta m \left( \frac{d^2r}{dt^2} \right)^2 = 2r \delta m \left( \frac{d^2r}{dt^2} \right) + \cdots
\]

Approximately, by delta differentation,

\[
2r \left\{ \delta m \left( \frac{d^2r}{dt^2} \right) \right\} \cong 2r \left\{ \delta m \frac{\Delta \left( \frac{d^2r}{dt^2} \right)}{\Delta t} \right\}
\]

and, by noting that by Newton’s second law of motion:

\[
f_t = \delta m \left( \frac{d^2r}{dt^2} \right),
\]

3. Derivation from the Spinning-Rod Quadrupole Formulation

An alternative derivation of Eq. (9) is as follows: From Eq. (1) of J. Weber (1964) one has for Einstein’s 1918 formulation of the gravitational-wave (GW) radiated power of a red spinning about an axis through its midpoint having a moment of inertia, \( I \), \( kg \cdot m^2 \), and an angular rate, \( \omega \), radians/s, [also please see, for example, Misner, Thorne, Wheeler (1973) in which \( I \) in the kernel of the quadrupole equation also takes on its classical-physics meaning of an ordinary moment of inertia]:

\[
P = 32G \frac{r^2 \omega^6}{5c^5} = G \left( \frac{I \omega^3}{5} \right)^{\frac{3}{5}} W,
\]

or, with \( I = r^2 m \) (\( r \) being the radius of gyration of the rod),

\[
P = 1.76 \times 10^{-52} (r \cdot m \omega^2) \omega^2 W
\]

where \( \{r m \omega^2\} \) can be associated with the magnitude of the rod’s centrifugal-force vector, \( f_{ce} \). Equation (11) is the same equation as that given for two bodies on a circular orbit by L. D. Landau and E. M. Lifshitz (1975) \( (I = m r^2) \) in their notation) where \( \omega = n \), the orbital mean motion, radians/s. The \( f_{ce} \) vector reverses every half period at twice the angular rate of the rod [and a \( f_{ce} \) component’s magnitude completes one complete period in half the rod’s period as in J. Weber (1964)]. Thus the GW frequency is \( 2(\omega/2\pi) \), where \( \omega \) is in radians/s. The change in the centrifugal-force vector itself (essentially a “jerk” when divided by a time interval) is a differential vector at right angles to the \( f_{ce} \) vector and directed tangentially along the arc that the rod ends move through. The differential change in, for example, the \( x \)-component of the change in centrifugal force, \( \Delta f_{cx} \), is \( f_{cx} \Delta \theta \) and the change in the \( y \)-component, \( \Delta f_{cy} \), is \( f_{cy} \Delta \theta \), where \( \theta \) is the central angle of the rotating rod in radians. By delta differentiation of \( f_{ce} = f_{ce} + f_{ce} \),

\[
f_{ce} \Delta f_{ce} = f_{ce} \Delta f_{ce} + f_{ce} \Delta f_{ce}
\]
THE BIG JERK

Excerpts from:


“Novel formulation of the quadrupole equation for potential stellar gravitational-wave power estimation” Astronomische Nachrichten / Astronomical Notes, Volume 327, 2006, No. 7, pp. 710-713.

Robert M L Baker, Jr.
3. Derivation from the Spinning-Rod Quadrupole Formulation

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$$P = 32G \frac{I^2 \omega^6}{5c^5} = G \frac{(I\omega^3)^2}{5 \left(\frac{c}{2}\right)^6} W,$$

or, with $I = r^2 m$ ($r$ being the radius of gyration of the rod),

$$P = 1.76 \times 10^{-52} \ (I\omega^3)^2$$

$$= 1.76 \times 10^{-52} (r \{rm\omega^2\} \omega)^2 W$$

(11)

where $\{rm\omega^2\}$ can be associated with the magnitude of the rod’s centrifugal-force vector, $f_{cf}$. 
Equation (11) is the same equation as that given for two bodies on a circular orbit by L. D. Landau and E. M. Lifshitz (1975) ($I = \mu r^2$ in their notation) where $\omega = n$, the orbital mean motion, radians/s. The $f_{ef}$ vector reverses every half period at twice the angular rate of the rod [and a $f_{ef}$ component’s magnitude completes one complete period in half the rod’s period as in J. Weber (1964)]. Thus the GW frequency is $2(\omega/2\pi)$, where $\omega$ is in radians/s. The change in the centrifugal-force vector itself (essentially a “jerk” when divided by a time interval) is a differential vector at right angles to the $f_{ef}$ vector and directed tangentially along the arc that the rod ends move through. The differential change in, for example, the $x$-component of the change in centrifugal force, $\Delta f_{e fx}$, is $f_{e fx} \Delta \theta$ and the change in the $y$-component, $\Delta f_{efy}$, is $f_{efy} \Delta \theta$, where $\theta$ is the central angle of the rotating rod in radians. By delta differentiation of $f_{efx}^2 = f_{efx}^2 + f_{efy}^2$, 

$$f_{ef} \Delta f_{ef} = f_{efx} \Delta f_{efx} + f_{efy} \Delta f_{efy}$$

and when one associates the components $\Delta f_{efx,y}$ with $f_{efx,y} \Delta \theta$ and, after dividing by $\Delta t'$ ($t'$ being spinning-rod time), and noting that $\Delta \theta/\Delta t' = \omega$, 

$$\frac{f_{ef} \Delta f_{ef}}{\Delta t'} = (f_{efx}^2 + f_{efy}^2) \omega.$$
Thus $\Delta f_{cf}/\Delta t' = f_{cf}\omega$; but $\Delta t' = \frac{1}{2}\Delta t$ since the period of the GW is half the period of the rod, so that:

$$2\frac{\Delta f_{cf}}{\Delta t} = f_{cf}\omega,$$

(14)

but $f_{cf} = \{rm\omega^2\}$ so

$$2\frac{\Delta f_{cf}}{\Delta t} = \{rm\omega^2\}\omega,$$

(15)

and substituting Eq. (15) into Eq. (11) yields

$$P = 1.76 \times 10^{-52} \left(2r \frac{\Delta f_{cf}}{\Delta t}\right)^2 W,$$

(16)

where $(2r\Delta f_{cf}/\Delta t)^2$ is the kernel of the quadrupole approximation equation and $\Delta f_{cf}/\Delta t$ is, again, the jerk.
In general:

GW Power = $1.76 \times 10^{-52} \ (2r\Delta f/\Delta t)^2 \ \text{W}$

Where $r$ is the radius from the GW focal point to the two opposed masses, $m$, and
- $\Delta f$ is the change, kg, in force over time $\Delta t$, s.

- $\Delta f/\Delta t$ is the “jerk”
GW GENERATED BY ORBITING MASSES AND BY TWO JERKING MASSES