Lessons for Energy Resonance HFGW Detector Designs from Mass Resonance and Interferometric LFGW Detectors

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Abstract. Many design and operations challenges have been met and surmounted by four decades of work on low frequency gravitational wave (LFGW) detection schemes such as mass-resonance bars, mass resonance spheres, and interferometers. This design and operations experience could be leveraged to benefit a new class of high frequency gravitational wave (HFGW) energy resonance detectors now in the early planning and design phases. Detection strategy lessons include the characterization of possible GW signal sources, the characterization of what constitutes a detection event, the role of various sources of 'Q' in effective signal amplification and filtering, and the sources of noise and how they can be reduced. Theoretical limits to sensitivity will also be summarized, including the quantum back action limit and how it may be avoided using quantum non-demolition, including the possibilities of squeezed states and even quantum coherence. Modeling, simulation, and processing lessons will also be reviewed, including the use of multiple detectors, delay histograms, statistical filtering, and the pitfalls of non-linear signal processing. Finally, technical management strategies will be reviewed, including collaboration options and data pooling decision strategies.

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INTRODUCTION

In this paper we analyze some of the design and operating strategies used in prior GW detector types and apply them to the emerging energy resonance type of GW detector. Detection strategies, signal to noise limits, signal processing approaches, and technical management strategies of prior art in gravitational detectors are all reviewed for applicability to the emerging class of energy resonance GW detectors.

GRAVITATIONAL WAVE (GW) DETECTOR TYPES

The development of three different types of gravitational wave detectors help delineate the three eras of gravitational wave detection attempts to date. First was the era of the mass resonance bar, also known as the "Weber bar," after its inventor Joseph Weber (see; Weber, 1969; Collins, 2004). This era ran from 1958 to well into the 1990's. Towards the end of this era resonant spherical mass resonators were also developed and designed, but were not widely deployed as working instruments. The next era has been that of the interferometric GW detectors, conceived in the 1960's but not well funded until the early 1990's, and now dominating the field of GW measurement attempts at the time of this writing. Finally, the latest era in GW detection, just now beginning, is that of the energy resonance detectors, which have yet to be built, but show promise from a theoretical point of view (Li et al., 2008 Li et al., 2008). These three types of detectors will be compared and contrasted in this section. In later sections lessons drawn from the first two types of GW detectors will be applied to the third type.

Mass Resonance Bars and Spheres as GW Detectors

Mass resonance bars as GW detectors were first built by Weber in 1962 (see; Collins, 2004), hence the name 'Weber bars.' Such an instrument is depicted in Figure 1. A massive cylinder of metal, usually Aluminum, is suspended in the middle of a vacuum chamber by means of a suspension wire hung from a frame on isolation mounts. Piezoelectric crystals are glued to the side of the cylinder to pickup any vibration present in the cylinder, after which it is sent through amplification and presented to the strip chart recorder. In the very early days of research there was no additional signal processing other than a human read interpretation. This understandably led to the realization that the notion of a detection event will require much more of a systematic approach, including the definition of a 'detection event' as a threshold crossing. Cryogenic versions later followed which improved the sensitivity by means of controlling the noise level through the reduction of thermal noise within receivers inside the system.



FIGURE 1. Mass Resonance Gravitational Wave Detector Design, commonly referred to as a 'Weber Bar' (Collins, 2004).

Dewar design is thus a consideration, not now just from the point of view of creating a vacuum to minimize vibration, but also from a thermal isolation point of view. Refrigeration can be supplemented using dilution refrigeration, which reduces temperatures to within 25 mK of absolute zero.

The next class of GW detectors is really a variation of the first. Spherical resonance GW detectors still depend on the measurement of GW interacting with dimension changes of physical objects, in this case the shape of coupled spherical cavities. Resonance is setup in each cavity that is nearly identical, and the energy exchange of coupled signal is monitored for disturbances due to GW (Bernard, 2001). Sensitivity can be improved by using superconducting spheres. Figure 2 for shows a detector of this type.



FIGURE 2. Xylophone Array of Spherical Resonance Gravitational Wave Detectors (Bernard, 2001).

While the frequency to which each spherical pair is sensitive is a rather narrow fixed frequency, various different frequencies can be detected by using an array of different sized spherical cavities, each tuned to different GW frequencies, which may be referred to as a "xylophone array." Presently there are no operational spherical resonance GW detectors that are known to the author. However a slightly different design of a shape deformation type GW detector that operates using contained energy is that of the circular waveguide, as developed by Ingley and Cruise (2001), has been intermittently operational over the past 7 years.

Interferometers as GW Detectors

Robert Forward built the first laser interferometer at Hughes Research Labs in 1972 (see; Blair, 1998). Currently most of the major GW observatories (really experiments) deployed worldwide are operating interferometers. This class of detector senses GW by measuring the relative length change along perpendicular arms of the interferometer by detecting and processing interference fringes presented by recombining the beams at the central intersection. A quadrupole deformation should shrink one arm and stretch the other. See Figure 3 for a schematic representation. LIGO, VIRGO, and GEO are all of this type. Because of the enormous size of these detectors, 4km per leg in the case of LIGO, and the number of passes back and forth that are made, which can be 75 or more times, time of flight of the laser's photons will limit the frequency of GW to which the detector is sensitive. Nevertheless there are many valuable lessons that may be drawn from the LIGO experience that would be applicable to higher frequency energy resonance detection systems.



FIGURE 3. The LIGO detector - an example of an Interferometric Gravitational Wave Detector Design (Blair, 1998).

As with any modern GW detector, the entire sensing assembly is enclosed within a cryogenic vacuum dewar. The center beam splitting and detection optics as well as both arm mirrors are all suspended by pendulum suspension systems, effectively isolating them from external vibrations. Residual vibrations are removed by "cold damping," a technique that actively mechanically compensates vibration to cancel it (Blair, 1998).

As was stated earlier, the same laser may bounce back and forth 75 to 100 times on each arm to maximize the effective measurement; this is known as power recycling (Blair, 1998.). In a similar manner, the resultant signal may be reflected and reinforced several times in a technique known as "signal recycling," and because mirror imperfections tend to cancel out after repeated reflections, this technique also tends to clean the signal, which is known as "wavefront healing". Both power and signal recycling improve quality factor, as will be seen in the quality factor section, and therefore improve Signal to Noise Ratio (SNR), as will be seen in the detection strategies section. This results in strain sensitivities on the order of 10^{-19} , with a goal of less than 10^{-22} once the use of squeezed quantum states is implemented in the laser. Squeezed states will be discussed in the quantum non-demolition section.

Energy Resonance Systems as GW Detectors

A newly emerging class of GW detectors is that of the energy resonant systems, first conceived by Gertsenshtein (1962). All GW detectors built to date have endeavored to measure GW by directly or indirectly measuring a change in length of a physical object (mass resonance) or physical distance (interferometers). In contrast, energy resonant systems are designed detect GW without measuring a mechanical length change, but instead by direct coupling between the metric oscillation that constitutes the gravitational wave, and an EM field oscillation, in a resonant manner. This detector class is in its formative stages and could benefit greatly by adopting lessons learned from the previous generations of GW detectors.

In the case of the detection process as envisioned originally by Gertsenshtein, the "synchro-resonance" mechanism arises naturally via the passage of a GW through a strong magnetic field, causing a resonating scatter of photons along the same axis. In the case of the Li-Baker detector, shown in figure 4, the strength of the synchro-resonance (also known in this case as "GW/EM super-heterodyne" by Becker, 2003) is aided by colliding an EM beam with a GW input of the same frequency, also in a strong magnetic field, to produce a synchro-resonance signal output, in this case perpendicular to both the EM/GW collision axis, and to the north to south axis of the magnetic field. Strain sensitivities on the order of 10^{-32} may be possible using existing technologies for HFGW in the microwave range.



FIGURE 4. The Baker-Li Detector – an example of an energy resonance GW detection system (Baker, Stephenson, and Li, 2008).

DETECTION STRATEGIES

It is a fair assessment that during the almost 50 years of GW detection attempts, millions of hours of analysis have been devoted to answering two central questions: first, what is it we are trying to measure, and second, how can we optimize the sensitivity to perform these predicted measurements. Lessons learned from this activity are summarized in the next sections.

Characterization of GW Signal Sources

What are the natural sources of GW? Natural sources may be summarized as consisting of the following astrophysical phenomena (Collins, 2004):

- <u>Burst sources</u>: as emerging from supernova, such as the highly controversial SN1987a event (Weber, 1969.) Waveforms can not be calculated in advance, so sightings on multiple detectors are needed to confirm.
- <u>In-spiraling objects</u>: in-spiraling neutron stars have a well known waveform already observed indirectly through energy loss in-spiraling black holes have a more complex waveform only recently calculated.
- <u>Continuous sources</u>: such as pulsars or any other rapidly spinning asymmetric astrophysical object. Here the advantage is that if the source frequency is known, the signal may be filtered and integrated over a long period of time.
- <u>Stochastic Background</u>: likely to be a combination of very faint and remote versions of all of the above, plus any gravitational radiation associated with the cosmic background, including any relic HFGW possibly generated during the big bang.

Searching for GW sources will therefore necessarily involve a diversity of different signal frequencies, bandwidths, and waveforms so that no one signal processing approach will be correct for all sources. It is therefore imperative that raw data be preserved until such time as all reasonable processing operations have been performed on that data without success.

The Definition of a Detection Event

The definition of a detection event is a foremost consideration, and lessons are studied both in terms of the threshold level and in terms of the statistics of exceeding that level. For a relatively close, strong source, a discrete event may be measured if it breaks a predetermined threshold. Most often the threshold is determined by selecting a threshold to noise ratio (TNR) that is judged to be statistically improbable. For Gaussian statistics this could be as low as 6 sigma for a single detector, or 3 sigma for 2 uncorrelated detectors. Two notes of caution are in order: first, filtering can affect statistics, so noise that may have started out with a Gaussian statistics in the raw phenomenology may very well turn out to be extremely non-Gaussian by the time a number of non-linear signal processing algorithms are applied. The second note of caution is that if data is shared with another detector team, it is important to understand any pre-processing they performed on the data, so that the same rules are applied to both sets of data. This is covered further in the data pooling section.

The detection of stochastic signals is more problematic. If the signals look random enough to be stochastic, then it stands to reason they will be very difficult to distinguish from noise, which is also stochastic. In a case like this the best one could hope for is to do a frequency spectrum analysis of the stochastic signature, to ensure that spectrum of known noise sources are discounted, and anything matching the known theoretical spectrum of GW is drawn out and highlighted, since it indicates a genuine GW signal.

The Role of Quality Factor in Detection

The effect of the quality factor inherent in the detection apparatus will be included in this area of lessons learned investigation. Quality factor (Q) may be understood as the quantification of how a signal is treated preferentially over noise. For instance, if a crystal of sapphire is struck with a vibration 100 msec in duration, but continues to ring for 1000 seconds as a result of this impact, then the crystal is said to have a $Q = 10^5$ because the short incident signal can be integrated over a much longer time to reap the benefit of the described signal persistence.

Power recycling and signal recycling also provide sources of Q in the sense that they boost usable signal over noise. Signal processing can also be a source of Q since processing gain is one way to boost signal to noise. Advantages and techniques of this process will be discussed further in the signal filtering techniques section, but the use of signal processing also has disadvantages and pitfalls.

Sources of Noise and Noise Reduction Techniques

In this section what is known about the sources of noise from prior detection attempts will be summarized, along with suggested mitigation techniques. The various noise sources typically encountered in GW detectors are listed below (Collins, 2004):

- <u>Amplitude noise</u> (of lasers) Also referred to as Shot noise, there is no way around this, as it is tied to the statistics of photons. It goes as the square root of the number of photons present in a laser sample.
- <u>Phase or Frequency noise</u> (of lasers) This is due to the fluctuations in the frequency of the laser, and can be mitigated by designing in a phase-locked-loop for lasers or other photon transmitters used within the detector system, such as the Gaussian beam in the case of the Li-Baker detector.
- <u>Mechanical thermal noise</u> Brownian motion of sensor components. Mitigation is to refrigerate the sensing apparatus to reduce thermal inputs.
- <u>Radiation pressure noise</u> Due to noise in the motion of reflecting elements due to radiation pressure variations when the power of the laser (or EM transmitter) varies.
- <u>Seismic noise</u> Probably not at factor for HFGW, this noise source can be removed by proper suspension mounting design.
- <u>Thermal gradient noise</u> imperfect heating of internal detector components; can be mitigated by good EM sense field uniformity, as in wavefront healing, and by insisting on good parts finish uniformity.
- <u>Cosmic ray noise</u> Noise caused by cosmic rays. Could put mirrors or other mechanism below ground if this source becomes a problem.
- <u>Gravitational gradient noise</u> Probably not a factor for HFGW, we should still be aware of any changes in mass distribution on the test floor as it disturbs the DC set point for gravitation.
- <u>EM field noise</u> EM fields may leak into receivers or amplifiers intended only for GW induced signals. Mitigation is to shield the entire sensing mechanism from EM fields. This could be a big problem for energy resonant systems given the enormous quantity of contained energy needed to couple to GW.

Vibration, acoustic noise, and crosstalk are also overriding concerns in any design, as is ensuring an electrically and thermally quiet an environment as possible for the GW sensing assembly.

SENSITIVITY LIMITS

Sensitivity limits will be analyzed in this section, integrating lessons on the limits of signal strength and noise level in "Signal to Noise Ratio," and quantum limits on measurement in "Quantum Back Action and the Standard Quantum Limit." Methods for overcoming some of these limits through the use of squeezed states and quantum coherence will also be summarized in "Quantum Non-Demolition: Squeezed States and Quantum Coherence."

Signal to Noise Ratio

Signal to noise ratio (SNR) is a critical performance parameter for all detectors, and GW detectors are no exception. There is not considered to be any detection until SNR = (S/N)(Q) > TNR, where S = signal strength, N = the root sum square of all noise components, Q = the quality factor, and TNR = the threshold to noise ratio that defines detection. Sources of quality factor Q were covered in the quality factor section. Increases in Q will increase SNR, but only if the signal strength does not fall faster than the quality factor increases. Anything that reduces noise will also increase SNR, but only if the limiting noise is reduced will the change in SNR be significant, since noise terms are root sum squared. Noise terms, along with mitigation strategies, were covered in noise reduction techniques.

Quantum Back Action and the Standard Quantum Limit

Among the many noise terms that limit SNR are the quantum noise limits. There are a variety of different noise terms that have the "quantum" label applied to them. First there is photon shot noise. For signals composed of photons this noise is the square root of the number of photons in the signal. There is also quantum jitter noise associated with the photons striking surfaces within the detector's sensing assembly. And finally, perhaps most

fundamentally, there is the quantum limit of the measurement process itself. Known as the "standard quantum limit" (SQL) and also as "quantum back action," it is the quantum mechanical impact of the measurement process on subsequent measurements. (For a quantification of this effect, see; Stephenson, 2009).

Quantum Non-Demolition: Squeezed States and Quantum Coherence

Is there a way around the SQL limit without "destroying" quantum mechanics? Answering this question is the genesis of the phrase "quantum non-demolition," or QND. QND may be achieved by any means that suppresses noise in a quantum state under measure by increasing noise in its quantum conjugate. In the case of sensing photons this may be achieved through the use of non-linear optics, a process known as "squeezing states." Improvements are in work to add the use of squeezed states to the LIGO lasers for an improvement in sensitivity from 10^{-19} to better than 10^{-22} strain sensitivity.

Another way to achieve QND may be through the use of quantum coherence. Quantum coherence is achieved when every atom of a material acts in unison for a result, such as a photon release, that is perfectly in phase. When applied to lasers or other coherent EM sources this is often also referred to as "super-radiance." This notion has had a controversial past, as it has been used by Weber to justify how it was possible for GW detectors to exceed their predicted sensitivity. Nevertheless it may still be worth investigating in the context of superconductors, which may be manipulated to present some of the characteristics of quantum coherence.

MODELING, SIMULATION, AND SIGNAL PROCESSING

Lessons from the modeling and simulation of prior GW detection systems and the design of the signal processing for those systems are summarized in this section.

Modeling GW Signal and Simulating Signal Gain

While it is true that simply because a device can be modeled and simulated does not guarantee it will work, so it is also difficult to imagine a device could be made to work that could not first be properly modeled. Indeed the very process of developing a model demands a certain level of understanding, which is how the notion of a model as a "thirsty tool" (Collins, 2004) arises. An end-to-end simulation of signals and noises has proven valuable on LIGO and validating the model against various portions of a design is a valuable exercise in understanding both the integrated "as built" instrument and its simulation. There are limits to what can be done in simulation – unknown noise sources come to mind – but the advantages of design trade capability and design driver discovery readily outweigh disadvantages of the additional development time and cost.

Modeling Noise Limits and Characterizing Instrument Noise

From section on sources of noise we now briefly revisit noise sources from the point of view of modeling noise limits within the system. Amplitude noise, phase or frequency noise, mechanical thermal noise, and radiation pressure noise are all stochastic in nature but have well described levels tied to design parameters within the sensing mechanism of the GW detector, and should therefore be straightforward to model. Modeling each noise source in the context of the signal present at that point in the signal path will lend context to each noise level, improving the analysis of which noise sources are design drivers. On the other hand, seismic noise, thermal gradient noise, cosmic ray noise, gravitational gradient noise, and EM field noise, while they need to be mitigated, especially if long integration times are being used, are probably not repeatable enough to effectively model within a simulated environment.

Combining Multiple Detectors and the use of Delay Histograms

Linear processing techniques such as multiple detector combination and delay histogram searches will be studied in this section. History has shown us that once a GW detector is operational it will invariably face the issue of false alarms. Merely setting a high threshold will not be enough. The question will invariably be asked: how do we know it was not just noise? Or interference from some non-GW source? For this reason GW detectors, if not built in pairs, have chosen to operate in teaming environments that permit processing at least two detectors together. Assuming we then have more than one data stream, how to we correlate, combine, and process them?

In the case of discrete events, multiple detectors, even detectors in completely different locations, can be combined via the mechanism of delay histograms. Referring to Figure 5, the signal from detector #1 can be compared with the signal in detector #2 by checking a range of possible delays between the signal of #1 and #2. With this technique it is more straightforward to combine peaks in the signals from discrete events. However, also note that because we are now checking over a wider range of times for a peak match, we are also increasing the probability of false alarms. Therefore it is imperative that the threshold for declaring a detection event be adjusted to compensate for the search gain, so that we do not permit this technique to downwardly skew the declarative threshold to noise ratio.



FIGURE 5. An example of a delay histogram when used to align detection events (Collins, 2004).

Coincidence analysis is a more general statement of histogram analysis, where multiple detector comparisons may be replaced by comparison with non-GW detectors, such as gamma rays bursts or neutrino detections. This is valid if phenomena that generate gamma ray bursts and neutrino bursts also generate GW bursts, which may be the case for example with supernova, and as previously mentioned is still being debated for the case of SN1987A.

Signal Filtering Techniques: Matched Filtering, Statistical Filtering

Another method of reducing false alarms is through the use of signal filtering. Filtering may be used to attempt the removal of false alarms through band stop filtering for noise spurs, through a running average of multiple samples, or through the use of recursive filtering. Certain signals could also be selected for enhancement, such as using a bank of matched filters for known signatures such as in-spirals. Long integration searches may be used to find CW sources such as pulsars.

Coherent processing may also be used for the case of stochastic signals, i.e. statistical filtering, such as will be the case for relic HFGW. In coherent processing the phase information of each of multiple detectors can be compared and searched for correlations. In the opposite approach, stochastic signals may be removed to more easily find discrete events, such as is done with optimum nonlinear filters. Multiple parallel processing chains could also perform all of these techniques in parallel, to check for all possible signals with all available means.

The Pitfalls of Non-Linear Signal Processing

Caution should be exercised whenever filtering is applied to data, especially if non-linear signal processing is used, since, without matching detection threshold changes, detection thresholds will not keep pace and false alarms will increase. Many of the false alarm problems that plagued Weber (see; Collins, 2004) were due to a lack of discipline in the area of detection threshold maintenance. It bears repeating that every single filter addition must be accompanied by a change in threshold to noise, with the goal of restoring false alarms to a statistical equivalent of a 6-sigma Gaussian event.

TECHNICAL MANAGEMENT STRATEGIES

Suggestions for technical management strategies are distilled from the history of GW detection in this section. Determining collaboration structure is covered in "Selecting Collaboration Levels," an exploration of data pooling options is included in "Data Pooling Protocols," and selecting detection declaration rules are delineated in "Detection Declaration Decisions."

Selecting Collaboration Levels

Experience from mass resonance detectors and interferometers have provided several levels of collaboration. At the higher level, a comparison of an "event list" may provide beneficial to all detector teams involved if there is some concern as to whether a detection event is a valid GW detection. If there was never a possibility of a false alarm, then there would be no need to collaboration. At a more detailed lower level, a comparison of the raw data by collaborating parties may prove to be beneficial to all parties involved. For discrete event analysis, collaboration allows delay histogram analysis and signal signature correlation analysis. For stochastic signals, collaboration allows for comparative coherent analysis. Without collaboration detector teams must operate multiple detectors on their own to have these methods at their disposal.

Data Pooling Protocols

Unfortunately, lessons from GW history also indicate that comparing raw data has its own pitfalls. First, the processing of raw data ex post facto runs the risk of encouraging the tailoring of signal processing to declare a detection. For this reason a number of techniques have emerged to help data analysts avoid temptation. The first of these was the "Louisiana protocol" for data correlation. In this protocol raw data was released but a delay was inserted that was unknown to the recipient partner, so that it was up to the recipient to pick the correct delay without any outside help. Additionally, the Louisiana protocol also included the insertion of "calibration pulses" that would provide false positives, so that if a correlation was found, the data recipient would have to check with LSU to determine whether they had a detection or a false correlation. The IGEC has reportedly (Collins, 2004) adopted a similar protocol, albeit somewhat simplified.

Detection Declaration Decisions

Detection declaration is the most important decision a GW detector team will have to make, so it should be treated with extreme caution. Once lost, the team's reputation will be difficult, if not impossible, to repair. Event declaration is important to regulate not just for the team, but for the entire field of experimental gravitational wave science.

CONCLUSION

Many lessons may be learned from the study of mass resonance and interferometric GW detectors that may be applied to the design and operation of energy resonance GW detectors. Chief among these lessons are strategies for the improvement of signal to noise, strategies for signal processing, and strategies for guiding collaborations, data pooling, and detection decision analysis.

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REFERENCES

- Baker, R. M. L., Stephenson, G. V. and Li, F., "Proposed Ultra-High Sensitivity HFGW Detector," in the proceedings of *Space Technology and Applications International Forum (STAIF-08)*, edited by M.S. El-Genk, AIP Conference Proceedings 969, Melville, NY, (2008), pp. 1045-1054.
- Becker, Robert E., "A Gravitational Archipelago," in the proceedings of the *First International HFGW Conference*, The Mitre Corp, HFGW-03-123, (2003).
- Bernard, P., Gemme, G., Parodi, R. and Picasso, E., "A detector of small harmonic displacements based on two coupled microwave cavities," *Review of Scientific Instruments*, Volume **72**, Number 5, May, (2001), pp. 2428-2437.
- Blair, D. and McNamara, G., Ripples on a Cosmic Sea, Addison-Wesley, Reading, MA., (1998).
- Braginsky, V. B., Grishchuk, L.P., Doroshkevich, A. G., Zeldovich, Ya. B., Noviko, I. D. and Sazhin, M. V., "Electromagnetic Detectors of Gravitational Waves," *Sov. Phys. JETP*, **38**, (1974), p. 865.
- Braginsky, V. B., and Rudenko, V. N., "Gravitational waves and the detection of gravitational radiation," *Physics Report* (Review section of *Physics Letters*), Volume **46**, Number 5, (1978), p. 165-200.
- Collins, H., Gravity's Shadow, University of Chicago Press, Chicago, (2004).
- Cruise, A. M. "An electromagnetic detector for very-high-frequency gravitational waves," *Class. Quantum Gravity*, Volume **17**, (2000), pp. 2525-2530.
- Gertsenshtein, M. E., "Wave resonance of light and gravitational waves," *Soviet Physics JETP*, Volume **14**, Number 1, (1962), pp. 84-85.
- Ingley, R. M. J. and Cruise, A. M., "An electromagnetic detector for high frequency gravitational waves," 4th Edoardo Amaldi Conference on Gravitational Waves, Perth, Australia, July, (2001).
- Li, Fang-Yu, Tang, Meng-Xi, Luo, Jun and Li, Yi-Chuan, "Electrodynamical response of a high energy photon flux to a gravitational wave," *Physical Review D*, Volume **62**, July 21, (2000), pp. 044018-1 to 044018 -9.
- Li, Fang-Yu, Baker, R. M.L Fang, Zhenyun, Stephenson, Gary V. and Chen, Zhenya, "Perturbative Photon Fluxes Generated by High-Frequency Gravitational Wave and Their Physical Effects," *The European Physical Journal C*, **56**, July, (2008), pp. 407-423.
- Rudenko, V. N. and Sazhin, M. V., "Laser interferometer as a gravitational wave detector," *Sov. J. Quantum Electron.*, Volume **10**, November, (1980), pp. 1366-1373.
- Stephenson, G. V., "The Standard Quantum Limit for the Li-Baker HFGW Detector," in these proceedings of the Space, Propulsion and Energy Sciences International Forum (SPESIF-09), edited by Glen A. Robertson, AIP Conference Proceedings, Melville, NY, (2009).
- Stratonovich, R. L. Optimum nonlinear systems which bring about a separation of a signal with constant parameters from noise, Radiofizika, 2:6, (1959), pp. 892-901.
- Weber, J. "Electromagnetic Coupled Detection of Dynamic Gravitational Force Gradients," *United States Patent #3*,722,288, (1969).