REFEREE 1

Space Technology and Applications International Forum (STAIF-2008)

Peer Review Report

(Please Return ASAP – Guidelines Follow this Form)

Title of Paper: Analyses of the Frequency and Intensity of Laboratory Generated HFGWs

Author: R. M. L. Baker, G. V. Stephenson, F.-Y. Li Log Number: 010__________

Reviewer Na[redacted]________ Date Reviewed: 07/26/2007______

Reviewer's Phone #:[redacted]__ Fax #[redacted]____________

Email: [redacted]______________________________

Review [redacted]klist:

YES_ Is the technical treatment plausible and free of technical errors?

YES_ Have you checked the equations?

NO__ Are you aware of prior publication or presentation of this work?

NO__ Is the paper too long?

YES_ Is the manuscript free of commercialism?

Please give detailed reviewer comments below. Your review must include at least one paragraph of commentary on the technical merits of the paper or else it cannot be used as a qualifying peer review. A separate sheet of paper may be used if more room is necessary.

Review:

I recommend that this paper be accepted.

The authors describe the theoretical premises for two different high-frequency gravitational wave generators. They define two different types of generators: laser-target and piezoelectric (Film Bulk Acoustic Resonators). The former type are energized by ultra-high intensity tabletop lasers while the latter type is energized by a myriad of magnetrons. The authors show that the theory underlying the two generator concepts faithfully reproduce the physics of the classical spinning rod, spinning dumbbell, or orbiting masses which produce gravitational waves according to Einstein's general relativity theory. The authors then show the physics and derive the fundamental engineering parameters.

REFEREE 2

I recommend that the paper be accepted for publication after my detailed comments are responded to.

Response to Reviewer's Detailed Comments 9-4-07
STAIF 2008 Submitted Paper Log No. 010:

"Analyses of the Frequency and Intensity of Laboratory Generated HFGWs"

by

Robert M L Baker, Jr., Gary V. Stephenson and Fangyu Li

"It should be mentioned that this paper is basically a description of the experimental set-up and does not present any data although that is inferred."

The revised Abstract and Introduction emphasizes that the paper is theoretical.

"Generally, the authors' analysis appears to be based upon the premise that the instantaneous HFGW generated is equal to |df/dt|. This is a vital point that the reader needs to assimilate quickly. If so, I think it would be worthwhile stating this clearly in a displayed equation together with justification and a reference, rather than (as at present) buried in the text a few lines before eqn 6."

Both in the original Abstract and original Introduction, it was stated that: "The size of the generated HFGWs is proportional to the absolute value of force change divided by the incremental time interval, that is the slope of the force versus time curve." It is, however agreed that this should be mentioned earlier in the MS and in equation form prior to the laser HFGW generator section.

Page 3, "the resulting HFGW...is about twice the frequency associated with the laser pulse". This statement is woolly because the only frequency directly associated with the laser pulse is the PRF, 10Hz. Therefore I see no reason to include it, particularly since a few lines later the MS states "There are two HFGW pulses per laser hit" which is much clearer.

"Similarly, it is not correct to state a few lines further "The frequency of the generated HFGWs would be..." since there are many components in the spectrum of the generated HFGWs. What the authors have calculated is the highest frequency component. It would be correct to refer to "the frequency" only in a case where the resultant HFGWs are approximately sinusoidal or at least repetitive over many cycles of the component calculated. In this case, the fundamental component is 10Hz."

The reviewer makes a very good point here since it is really the delta t of the instantaneous HFGW burst or pulse that is important – not some inferred "frequency." We have now utilized the terms "cycle time" and/or "HFGW pulse δt" and avoided the term frequency as much as possible.

"Page 4, what is meant by "approximately an elongated Gaussian"? As far as I can see, the point is that each pulse is the derivative of half of a Gaussian. This shape is itself *not* a Gaussian, so why claim that it is, even approximately? A Gaussian is a Gaussian, whether "elongated" or not. I assume the authors mean the shape in time here; or do they mean spatial shape? I would expect the two are equivalent, but maybe not."

Well, this get a little involved. According to Ruxin Li (who is associated Director of the Shanghai Institute of Optics and Fine Mechanics and essentially runs the Chinese Ultra High Intensity Laser program) the laser pulse is only approximately Gaussian and its pulse intensity I roughly follows

$$I = a \exp(-k[t-t_0]^2) \qquad (1)$$

Roughly because the "Bell" is somewhat flattened at the top. The slope (derivative) of this curve (i.e., Eq. (1)) as a function of time returns a factor times a Gaussian curve. But this is just rough analysis and basically the laser target force must be calibrated during an experimental trial. We have now tried to get this rather elaborate concept across with a minimum of verbiage – a rather difficult task

"The Gaussian beam...of the HFGW detector" needs some preparation... the structure and operation of typical HFGW detectors has not been summarized in this MS, so this paragraph comes out of the blue without any further signposting or explanation. I *think* what the authors mean is that the acceptance plot of the detector need not match exactly the radiation plot of the generator...?."

The authors made the mistaken assumption that the readers of this paper would be familiar with the handful of papers describing the inverse Gertsenshtein-effect HFGW detectors and a couple of papers concerning HFGW generators presented at former STAIF get togethers. Clearly this is a very hubris assumption and very few knowledgeable GW scientists have ever read these papers – and why should they have? Our objective is not to be an elite "Club," but to attempt to get our ideas out in the general scientific arena in a reasonable form for evaluation! It makes it a bit difficult, but we have now attempted to explain the HFGW detectors of interest without being verbose.

"The justification for the analysis used in the case of the magnetron-driven FBARs is very difficult to understand. Is the point of this explanation that the oscillating FBARs are equivalent to an *oscillating* dumbbell rather than to a rotating dumbbell? If so, can this be stated, rather than the current opaque comments about snapshots?"

Here again our egotism in believing that the scientist readers have studied a couple of the papers on FBAR HFGW generation, presented at previous STAIF meetings, has gotten in our way of being clear. We had attempted to remedy this with a minimum of words in describing the FBAR HFGW generator.

"One line after fig 5, the text states "delta t corresponds to half of a Magnetron's EM wavelength". I suppose the authors mean "cycle period" rather than "wavelength"; but, even so, this is not what is drawn in fig 5 where delta t is clearly shown as the full cycle period of the excitation."

We have added to the figures to show a distinction between the δt cycle time of the HFGW and the cycle time of the force Δt created by the FBARs. Here by "cycle" we mean a complete sequence – that is starting at one point and returning to that point and starting again. We thought that would be clear looking at the figures and reading the text. Allow us to digress a little here. On page 356 of Landau and Lifshitz there is an interesting student problem (solved – thank heavens). It involves two masses on orbit. As they move opposite each other on a circular orbit they trace out a radiation pattern shaped like a figure "8" centered in between them, but with polarization lined up with the line between the two masses. The figure "8" produces a peanut-shaped figure of revolution radiation pattern. The cycle for the orbit (period) is completed each time the masses return to their original location and the cycle for the GW radiation is completed each time the figure of revolution is completed. It is completed each HALF orbital period. Thus two radiation patterns are produces each single orbital period. GW scientists say "The GW frequency is twice the orbital frequency." In the FBAR case two complete GW radiation pulses (we call them cycles) are completed during each FBAR cycle.

"A few lines later in this para, the mass of the FBAR membrane is given as 30ng; this is equivalent to 30 x 10^-9g = 30 x 10^-9 x

10^-3kg = 30 x 10^-12kg, not 3 x 10^-12kg as stated. At least one of these values must surely be incorrect."

You are right it was a typo: 3 is now replaced by 30.

"Conclusions, line 5, "if it is "followed" by the EM detector" is woolly. Do the authors mean that the predominant components in the radiated frequency spectrum must correspond with the passband of the detector, as in any tuned transmit/receive system? (In FM systems, there are sidebands transmitted out to zero and infinite frequency, but acceptable performance is obtained by having a receiver passband tailored to the bulk of the radiated components, rather than to the frequency deviation alone.)"

We have attempted to clarify this.

Appendix A line above Eq. (4a) there was an error – it is $h_{22} = -h_{33}$ as per page 346 of Landau and Lifshitz 14th line up. Rest follows correctly.

"Minor typos:

"Abstract 3 lines from end, period needed after "HFGWs" Done

"Introduction 3 lines from end, confused syntax because the subject of the sentence is HFGWs and also the laser-target generator; but these aren't parallel constructions so the rest of the sentence ("is especially well suited to...") can't apply to both   Done

"eqn 5, I think a negative sign is omitted from the units "s^1". Done

"Actually the units are irrelevant and unnecessary here anyway, since the units are contained in the algebraic quantities We still would like to carry some of them for clarity.

"2 lines after eqn 5, "as" should be deleted; the basic phrase structure is "the largest force possible", not "the largest force as possible" Done

"Li's and Woods's comments should be given as references to private communications (carry-over from previous version)Done

"Period needed at end of Li's quotation Done

"Poor style page 5, ""no it is a series..." (carry-over from previous version) Done – but what is this "Carry Over" – we did not receive such detailed reviewers comments previously -- just suggestions on how the paper needed considerable revision (no Fourier series, etc.) and some guidelines – very strange; maybe the original detailed reviewers comments were not attached or garbled in transmission?

"Line immediately after fig 5, there appears to be a stray period after GHz" Found it!

m/m Done

Our thanks go to the reviewer.

Analyses of the Frequency and Intensity of Laboratory Generated HFGWs

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Abstract. The theoretical concept underlying two laboratory high-frequency gravitational wave or HFGW generator designs or devices is presented. The generators are of two types: laser-target and piezoelectric or Film Bulk Acoustic Resonators (FBARs). The laser-target device is energized by ultra-high-intensity lasers and the FBAR device is energized by a myriad of Magnetrons. Such HFGW generators emulate the classical spinning-rod (or dumbbell) or orbiting-mass GW generating systems that are discussed by Baker (2006). The laboratory HFGW generators emulate these classical systems by utilizing an impulse or acceleration change over a very brief time interval that can be considered to be a "snapshot" or brief time-span picture of the classical systems. The laser targets or FBAR vibrational membranes undergo the force change captured by this "snapshot," but there is a small variation in the force with time, or first time derivative of force, over the incremental time period of the snapshot. The paper theoretically examines the force waveform or wave shape as well as the HFGW waveform generated during the infinitesimal time. It is concluded that a synchro-resonance (inverse Gertsenshtein effect) detector, such as proposed by Li, Baker and Fang (2007), works best if its EM detection beam (a Gaussian beam), which is an essential element of that HFGW detector, replicates the GW frequency, speed and waveform of the of the laboratory generated HFGWs. For other detectors, such as electromagnetic, resonance cavity or solid state e.g., "large crystal" (phonon producer), the waveform serves as a template for the expected signal. The size of the generated HFGWs is proportional to the absolute value of force change divided by the incremental time interval, which is the slope of the force versus time curve. A generalized design-parameter relationship for a HFGW laboratory generator is derived

Keywords: Laser, Microwaves, Gravitational Waves**,** High-Frequency Gravitational Waves.
PACS: 04.30.–w, 04.30.Db, 04.80.Nn, and 42.62.-b.

INTRODUCTION

The theoretical concept underlying two laboratory high-frequency gravitational wave or HFGW generator designs or devices is presented. The generators are of two types: laser-target (Baker, Li and Li, 2006) and piezoelectric or Film Bulk Acoustic Resonators or FBARs (Baker, Woods and Li, 2006). The reader is encouraged to be familiar with these two papers for background information. The laser-target device is energized by ultra-high-intensity lasers and the FBAR device is energized by a myriad of Magnetrons. Such HFGW generators emulate the classical spinning-rod (or dumbbell) or orbiting-mass GW generating systems that are discussed by Baker (2006). The laboratory HFGW generators emulate these classical systems by utilizing an impulse or acceleration change over a very brief time interval that can be considered to be a "snapshot" or brief time-span picture of the classical systems (Baker, 2000). The laser targets or FBAR vibrational membranes undergo the force change captured by this "snapshot," but there is a small variation in the force with time, or first time derivative of force, over the incremental time period of the snapshot. The paper theoretically examines the resulting force waveform or wave shape as well as the HFGW waveform generated during the infinitesimal time and concludes that a synchro-resonance (inverse

Gertsenshtein effect) detector, such as proposed by Li, Baker and Fang (2007), works best if its EM detection beam (a Gaussian beam), which is an essential element of that HFGW detector, replicates the GW frequency, speed and waveform of the of the laboratory generated HFGWs. For other detectors, such as electromagnetic, resonance cavity or solid state e.g., "large crystal" (phonon producer) e.g., as described by Grishchuk (1977; 1988; 2007), the waveform serves as a template for the expected signal. One detector considered herein involve a strong electromagnetic (EM) beam (either a laser or microwave) whose cross sectional energy variation is Gaussian (hence a "Gaussian Beam" or GB). A strong static magnetic field crosses this GB at the center of this particular detector and if HFGWs move parallel with the GB and have synchro-resonance, that is same speed and waveform (or "frequency"), then detection photons are generated and when they are sensed at an EM receiver the HFGWs are thereby detected. In both of the HFGW generators considered the size of the generated HFGWs is proportional to the absolute value of force change divided by the incremental time interval, that is, the absolute value of the slope of the force versus time curve.

ANALYSIS FROM SPINNING-ROD OR DUMBBELL VIEWPOINT

Let us consider a dumbbell-like spinning rod exhibiting a radius of gyration (essentially half of the dumbbell's length) r meters, a change in the centrifugal force vector (perpendicular to the centrifugal force vector itself and tangent to the path of the rod's ends), Δf, Newtons over an incremental time change, Δt seconds. According to Eq. (9) of Baker (2006) such a rod rotating at a constant rate or frequency will generate gravitational waves (GWs) having a constant power, P, given by

$$P = 1.76\text{x}10^{-52}\,(2r\,\Delta f/\,\Delta t)^2 \quad \text{W}, \tag{1}$$

which is derived directly from the classical equation for the power generated by a spinning rod, for example, given by Misner, Thorne, and Wheeler (1973),

$$P = 32GI^2\omega^6/5c^5 \text{ W}, \tag{2}$$

where G is the universal gravitational constant = $6.67432\text{x}10^{-11}$ $m^3/kg\text{-}s^2$, I is the rod's moment of inertia $kg\text{-}m^2$, ω is the rod's angular rate radians s^{-1} and c is the speed of light = $2.998\text{x}10^8$ ms^{-1}. The near field concerning the relative size of r and the HFGW wavelength and boundary conditions should be analyzed also using conventional general relativity (GR) theory as should the triple time derivative of the quadrupole formulated as Eq. (1). The rotating rod or dumbbell produces a constant amplitude gravitational wave having a moving plane of polarization as the dumbbell rotates. The moving plane of the polarization is perpendicular to the plane of the rotating dumbbell and includes the longitudinal axis of the dumbbell (the rotating line between the two radiating masses). The spinning dumbbell also produces a peanut-shaped GW radiation pattern, for example as derived by Landau and Lifshitz (1975) and discussed by Baker, Davis, and Woods (2005), whose axis is along the axis of dumbbell's rotation and centered midway between the two dumbbell masses. This pattern is a figure of revolution developed as a figure "8" shaped radiation pattern rotates as the dumbbell revolves. Each dumbbell revolution sweeps out two such peanut-shaped radiation patterns so that the frequency of the GW is just twice that of the rotating dumbbell. The HFGW flux of the generator F_{GW} moving parallel to the GB (in either of the peanut-shaped radiation pattern caps intercepted by a cone having a ten degree or 10^o semi-vertex angle), from Eq. (10) of Baker, Davis and Woods (2005), is given by

$$F_{\pm 10}{}^o = P\,2.54\,(0.282/D)^2 = F_{GW} \;\; \text{W m}^{-2}, \tag{3}$$

where D = the distance in either direction from the GW focus at the center of the HFGW detector..

Let us now imagine a snapshot of the spinning system. That is, we look at the dumbbell and GW system over a brief time span, Δt. The $\Delta\boldsymbol{f}$ vectors, having scalar components Δf, are the change in centrifugal force at each dumbbell mass. Of course one cannot have a perfectly instantaneous picture of the system – it will be over the infinitesimal time interval Δt as shown in Fig. 1 for a typical spinning dumbbell, where the force f could be a centrifugal-force component e.g., f_x .

Given the HFGW-generator's flux, F_{GW} Wm^{-2}, from Eq. (3), we have from Appendix A:

$$A = 1.28\text{x}10^{-18}\, F_{GW}^{\,1/2}/\nu_{\mathrm{GW}} \;\; \text{m/m}, \tag{4}$$

where A is the amplitude of the actual HFGW signal (expansions and contractions of spacetime or dimensionless spacetime strain) and ν_{GW} is the GW frequency for a spinning rod having angular rate ω rad s^{-1} , given by:

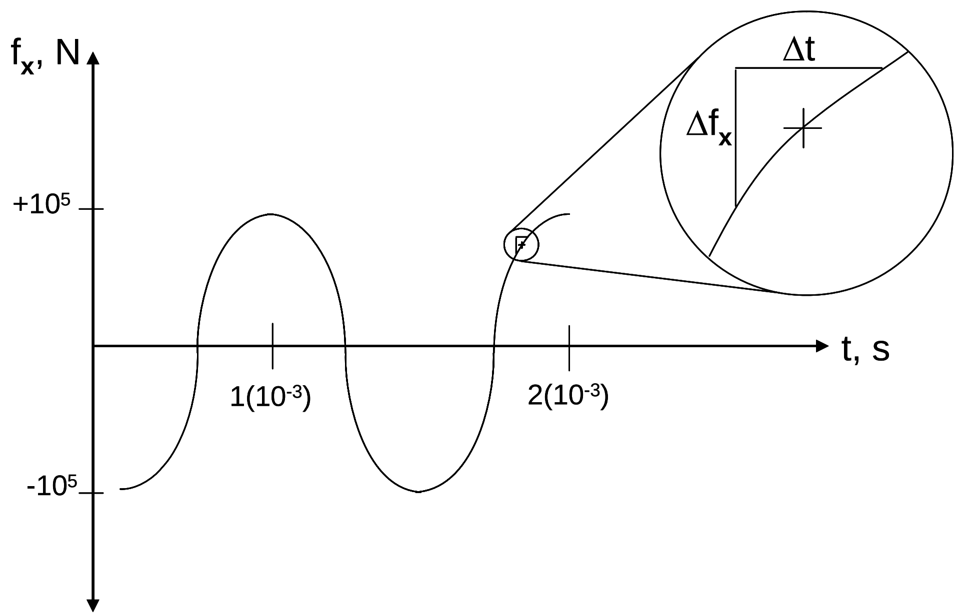


FIGURE 1. Change of the x-Component of Centrifugal Force with Time for a Spinning Rod or Dumbbell.

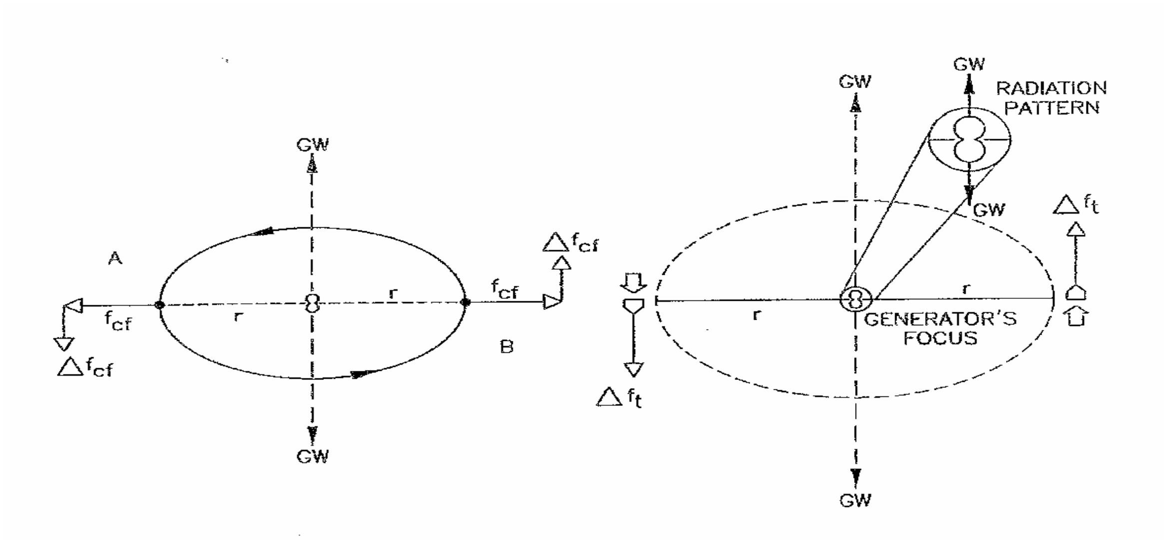
$$\nu_{\mathrm{GW}} = \omega/2\pi \;\; \text{Hz or s}^{-1} \tag{5}$$

In which ν_{GW} is the frequency of the HFGW distortions of the fabric of spacetime. The objective is to create the largest possible change in force, Δf, over the given time interval, Δt, in order to achieve the largest HFGW amplitude from Eq. (1). It is noted that the HFGW power given by Eq. (1) is proportional to the square of $\Delta f/\Delta t$ (the slope of the f versus t curve), whereas the HFGW amplitude in Eq. (4) is proportional to the square root of the power. Thus essentially we have a square root of a squared quantity or simply an absolute value of the slope, $\Delta f/\Delta t$ or:

$$A \sim |\, \Delta f/\Delta t|. \tag{6}$$

SITUATION FOR A LASER HFGW GENERATOR

Consider the Laser HFGW Generator described in Baker, Li, and Li (2006) and diagrammed in Fig. 2. In this case the change in force (analogous to the change in centrifugal force $\Delta \boldsymbol{f}_{cf}$ or a scalar component Δf of the centrifugal force, $\Delta f_{\mathrm{cf},}$, shown on the left side of Fig. 2) is the impulsive vector force $\Delta \boldsymbol{f}_t$, which acts on each of the laser targets, or gravitational-wave radiators, when the laser pulses strike (shown on the right side of Fig. 2). It is analogous to or a proxy for the change in centrifugal force for the dumbbell. The plane of polarization is perpendicular to the plane of the $\Delta \boldsymbol{f}$'s and includes the line between the two laser targets. From Baker, Li and Li (2006) $\Delta t = 3.39\text{x}10^{-14}$ s and $\Delta f = 1.5\text{x}10^{5}$ N. Let us select two values of r: the laboratory value of 10 m and the lunar-distance value of $4x10^8$ m. In each of these cases the waveform of the force as a function of time acting on each laser target as exhibited in Fig. 3. The waveform in Fig. 3 is



based upon Ruxin Li's (2007) remarks that the figure "…is very reasonable, because the rise time of the X-ray laser's target motion is very complicated and the real laser pulse shape is like Gaussian." Thus the ray

FIGURE 2. The Dumbbell and Laser-Pulse HFGW Generators Exhibiting the Δ*f*.

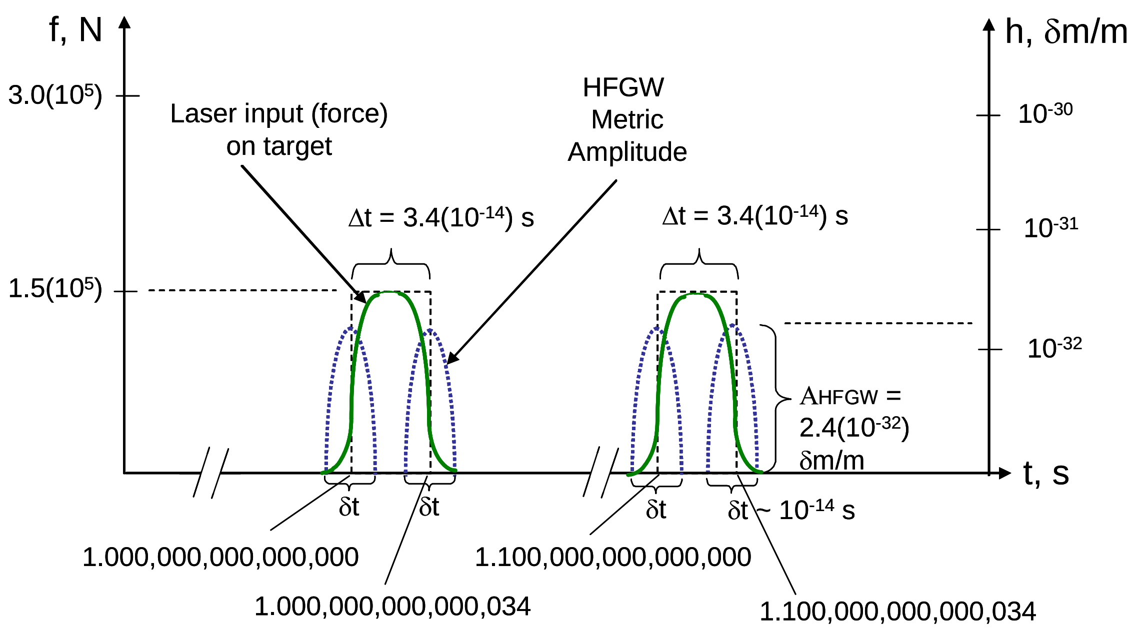


FIGURE 3. The Force Change and Generated HFGW Waveforms for a Laser-Pulse HFGW Generator.

resulting HFGW, whose amplitude is the absolute value of the slope of the force-versus-time wave shape, exhibits a HFGW burst time, δt, that is less than the Δt associated with the duration of the laser pulse and depends upon the rise time of the laser pulse. There are two HFGW pulses per laser hit generated by the large slope of f at each edge of the approximately Gaussian laser-target force waveform as shown in Fig. 3. The true shape of the force on the laser target as a function of time is defined by calibrating the laser systems during the experimental trials. As mentioned it is approximately Gaussian or a "bell curve," but with a flattened top. The slope or time derivative of this curve is also bell shaped, but rather elongated and, prior to laser calibration, can only be approximately described as in Fig.3. The Gaussian beam (GB) of the HFGW detector for the laser HFGW generator need not have exactly this HFGW shape as long as it overlaps the generated HFGW waveform. The GB's radiation is tailored to the bulk of the radiated components of the HFGWs to be detected. The laser GB for the HFGW detector should exhibit an approximate Gaussian shape and be in synch with the leading edge of the energizing laser pulse. The lasers (target energizing and GB) should also have the same polarization, but their frequency is not as important since the laser-target mass force change and the resulting HFGW gravitons are defined by their pulse time δt Thus the following calculation for the HFGW ripples in spacetime having amplitude, A, using Eq. (4) with $\nu_{GW} \approx 1/\delta t$ and the parameters given in Baker, Li and Li (2006) is :

$$A(r = 10\text{ m}) \sim 2.4\text{x}10^{-32}\text{ m/m}, \tag{7a}$$

and

$$A(r = 4\text{x}10^{8}\text{ m}) \sim 2.1\text{x}10^{-25}\text{ m/m}. \tag{7b}$$

SITUATION FOR A MAGNETRON-FBAR HFGW GENERATOR

Consider the Magnetron-FBAR, Piezoelectric-Crystal HFGW Generator described in Woods and Baker (2005) and Baker, Woods and Li (2006) and shown schematically in Fig. 4. In this case the change in force (analogous to or a proxy for the change in centrifugal force for the dumbbell) Δf is the impulsive force that acts on each Film Bulk Acoustic Resonator's (FBAR's) vibrating membrane when energized by a Magnetron's electromagnetic (EM) microwaves. When first considered it appears that this HFGW generation concept is quite different from the laser generator: The laser HFGW generator produces HFGW pulses whereas the Magnetron-FBAR generator produces continuous HFGWs. But in detail they are the same. The laser pulses are considered to be a series of "snapshots" taken at the laser pulse rate or ten times a second, whereas the Magnetron-FBAR generator can be considered to produce a *continuous series of snapshots.* It should be recognized, however, that this series is *not* a "moving picture" of the rotating dumbbell, which is being emulated; rather it is a series of totally independent snapshots each having its own Δt*, but representing again and again the same snapshot of the rotating dumbbell* – at the *same* point of the rotating motion! The plane of the polarization remains fixed being defined by the fixed line between the FBAR clusters and the fixed plane perpendicular to the plane containing the two $\Delta \boldsymbol{f}$ vectors (each $\Delta \boldsymbol{f}$ represents the net force change of a cluster of FBARs acting in concert). In Fig. 4 *13* and *14* are the Magnetron-FBAR clusters, *15* are the Magnetron energizers, *16* are the collections of FBARs (i.e., wafers), *17* are the Δfs, *18* and *19* are the oppositely directed summation of force changes, *20* is the imaginary circle or orbit being emulated by the HFGW generator, *21* is the imaginary plane of that of that circle or orbit, *22* is the peanut-shaped HFGW radiation pattern and *23* is the HFGW focus or origin of the radiation pattern. If instead of spherical clusters the Magnetrons and n FBARs were lined up on parallel lines (or tracks) a few kilometers in length, as proposed by Dehnen and Romero-Borja (2003), then as they prove the flux or intensity increases as n^2 and $\Delta f = n\Delta f_i$. In order to obtain generalized estimates of the performance of the linear Magnetron-energized-FBAR array we note that Eq. (1) can be phrased in terms of HFGW frequency, ν_{GW}:

$$P(\text{r}, \Delta f, \nu_{GW}) = 1.76\text{x}10^{-52}\,(2\text{r}\nu_{GW} n\Delta f_i\,)^{2}\text{ W}. \tag{8}$$

For a long linear array of FBARs the flux is not given by Eq. (3), but by $F_{GW} = P/\delta\text{A}$, where the reference area δA equal to some factor, k of the area of the diffraction pattern having diameter (to the first GW diffraction ring) d. We set $d = 1.2\,\lambda_{GW}$ in which λ_{GW} is the GW wave length = c/ν, c being the speed of light. So that $\delta\text{A} = \pi d^2/4 = \pi\text{c}^2/4\nu_{GW}^2$. Thus when we include the number of elements n in the HFGW generator's linear array and produce the needle radiation pattern, the HFGW flux is given by

$$A = 1.28\text{x}10^{-18}\{1.76\text{x}10^{-52}(2\text{r}\nu_{GW}\, n\Delta f_i\,)^2/[k\,\pi\text{c}^2/4\nu_{GW}^2]/\text{n}^2\}^{1/2}/\nu_{GW}\text{m/m}. \tag{9}$$

We combine Eq. (9) with Eq. (4) and obtain the generalized design-parameter relationship (or figure of merit) for a HFGW piezoelectric-crystal laboratory generator as:

$$A \text{ is proportional to r } \nu_{GW}\, \Delta f_{\text{i}}\, n^2. \tag{10}$$

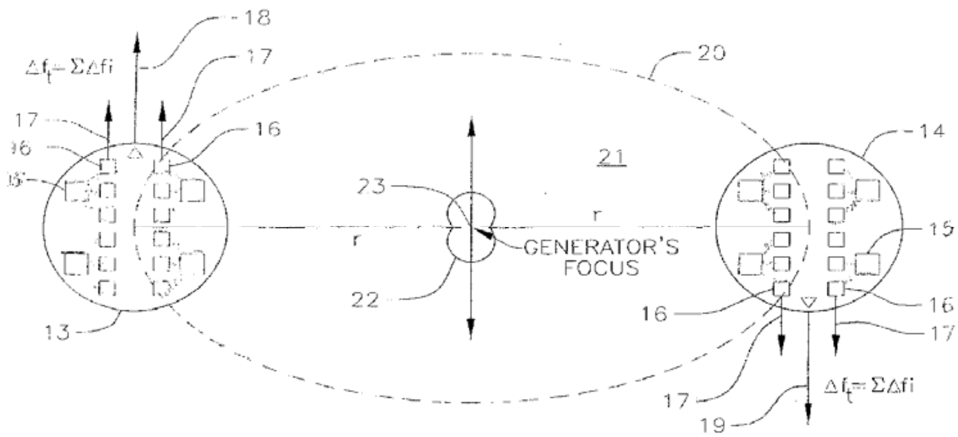


FIGURE 4. Magnetron Energized FBARs HFGW Generator Exhibiting the Δf.

The wave form in Fig. 5 for the FBAR's is based upon the remarks of R. Clive Woods (2007): "FBARs are driven by magnetrons which give a sine wave excitation. Also, they are highly resonant and will filter out any higher harmonics present in an imperfect magnetron drive."

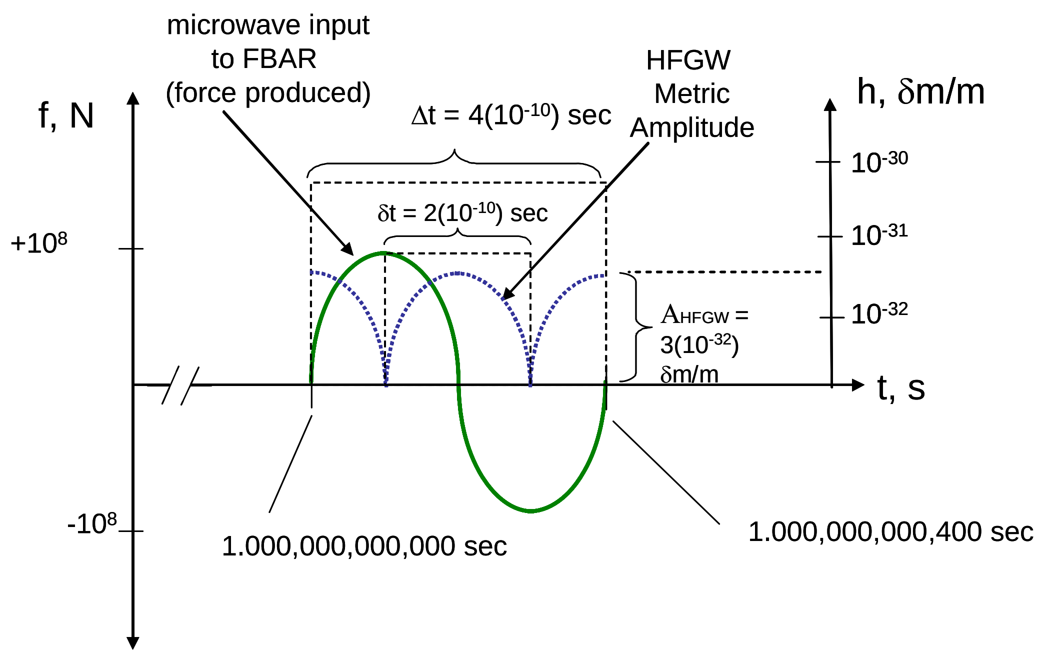


FIGURE 5. The Force Change and Generated HFGW Waveforms for the Vibrating Membranes of Magnetron-Energized FBARs.

The frequency of the energizing Magnetrons is 2.45 GHz = $2.45\text{x}10^9$ Hz. The HFGW cycle time δt corresponds to half of a Magnetron's EM cycle time Δt =1/$2.45\text{x}10^9$ = $4\text{x}10^{-10}$ s since, like the laser-generated HFGW, the figure "8" radiation pattern is produced every half cycle. Thus the generated HFGW cycle time δt = 1/2x$2.45\text{x}10^9$ = $2\text{x}10^{-10}$ s. From Baker, Woods, and Li (2006) the total Δf of each cluster of FBARs = $4\text{x}10^8$ N and from Woods and Baker (2005) the total number of FBARs in each cluster is 30,000 FBAR wafers x 6,000 FBARs = $1.8\text{x}10^8$ = n FBARs in each cluster and each FBAR exhibits a force change of 2 N so about $4\text{x}10^8$ N in sum for each cluster. From Woods and Baker (2005) the mass of the FBAR vibrating membrane is 30 ng or $30\text{x}10^{-12}$ kg. Let us select the laboratory value of r = 300 m from Baker, Woods, and Li (2006) and from that same reference we find P = $2.4\text{x}10^{-10}$ W and the HFGW flux F_{GW} = $1.4\text{x}10^{-8}$ Wm^{-2}. The small change in HFGW intensity over the Δt interval is two waves as shown in Fig. 5 (absolute value of the slope or derivative of the sinusoid force waveform). Assuming the alignment of the FBARs on parallel linear tracks the Δf_i accumulate as n^2 and $A_{\text{HFGW}} \sim (3\text{x}10^{-32})\,(1.8\text{x}10^8) = 4\text{x}10^{-24}$. Not all of the FBARs might be in phase on the wafers so the A may be less, e.g., ~ 10^{-26} δm/m. The microwaves of the GB HFGW detector's beam would only need to overlap the waveform of the generated HFGWs and could be a series of rectified sine waves (or EM pulses) having half the Magnetrons' cycle time, δt, and 90^o out of phase with the FBARs. For other HFGW detectors the waveform could serve as a template.

For the laboratory case of r = 300 m using the parameters of Baker, Woods and Li (2006): Eq. (4) yields:

$$A = 1.28\text{x}10^{-18}\,\text{x}(1.4\text{x}10^8)^{1/2}/4.9\text{x}10^9 = 4\text{x}10^{-32}\ \text{m/m to} \sim 10^{-24}\text{m/m}. \quad (11a)$$

For the lunar-distance case r = $4\text{x}10^8$ m with P = 420 W and F_{GW} = $2\text{x}10^5$ Wm^2 (Baker, Woods, and Li, 2006) if Eq. (1) holds, then Eq. (4) yields:

$$A = 1.28\text{x}10^{-18}\,\text{x}(2\text{x}10^5)^{1/2}/4.9\text{x}10^9 \sim 10^{-25}\ \text{m/m to} \sim 10^{-17}\ \text{m/m}. \quad (11b)$$

CONCLUSIONS

The change in centrifugal force of a dumbbell, emulated by HFGW generators, is perfectly "smooth." That is the change in each centrifugal-force component is perfectly uniform, as shown in Fig.1, as the dumbbell masses revolve. This is in contrast with the laser targets (Fig. 3) and the FBARs (Fig. 5) that represent a snapshot of the emulated dumbbell during the brief time interval. Here there is a unique waveform to the HFGW generated having half the cycle time of the force waveform. This HFGW waveform is "followed" by the GB detector's Gaussian beam that is the GB's radiation is tailored to the bulk of the radiated components of the HFGWs. For other HFGW detectors the waveform could serve as a template. In the case of the Magnetron energized FBAR generator the FBARs should be aligned along parallel tracks and kept in phase to allow for a n^2 accumulation of GWs. It is also concluded that the results given should be placed in the context of conventional GR theory or compared with the previous work by Dehnen and Romero-Borja (2003).

ACKNOWLEDGMENTS

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APPENDIX A

This Appendix is abstracted from Baker, Woods and Li (2006). From Eq. (107.11) of Landau and Lifshitz (1975), and considering the Transverse Traceless Gauge (i.e., TT Gauge), the nonzero quantities of the metric perturbation h_{ij} for the GW propagating along the x-axis will be h_{23} and $h_{22} = -h_{33}$. In this case the energy flux of the GW is:

$$F_{gw} = ct^{01} = \frac{c^3}{16\pi G}\left[\dot{h}_{23}^2 + \frac{1}{4}\left(\dot{h}_{22} - \dot{h}_{33}\right)^2\right], \tag{1a}$$

where by definition $\dot{h}_{ij} = \partial h_{ij}/\partial t$. Notice that Eq. (1a) must contain differentiation of h_{ij} to time (see, e.g., Misner, Thorne and Wheeler, 1973, Eq. (35.27)). Clearly, for the assumed *monochromatic* wave of frequency ω, which propagates along the x-axis, the general form of h_{ij} is:

$$\mathrm{h}_{ij} = A\exp\left[i\left(\omega t - kx\right)\right]. \tag{2a}$$

Thus the partial derivative with respect to time is:

$$\dot{\mathrm{h}}_{ij} = i\omega A\exp\left[i\left(\omega t - kx\right)\right] \text{ and } \left(\dot{\mathrm{h}}_{ij}\right)^2 = \omega^2 A^2 \tag{3a}$$

where A is the amplitude of the GW. Because $h_{22} = -h_{33}$, Eq. (1a) can be reduced to:

$$F_{gw} = \frac{c^3}{16\pi G}\left[\left(\dot{h}_{23}\right)^2 + \left(\dot{h}_{22}\right)^2\right]. \tag{4a}$$

In general, we can set $h_{23}=h_{22}$, in this case, from Eqs. (3a) and (4a), we obtain:

$$F_{gw} = \frac{c^3}{8\pi G}\left(\dot{h}_{22}\right)^2 = \frac{c^3}{8\pi G}\omega^2 A^2. \tag{5a}$$

Finally, solving for A one finds:

$$A = \left(\frac{8\pi G F_{gw}}{c^3\omega^2}\right)^{\frac{1}{2}} \approx 1.28\times 10^{-18}\ \mathrm{F}_{\mathrm{GW}}^{\frac{1}{2}} \Big/ \nu_{\mathrm{GW}}. \tag{6a}$$

This is Eq. (4) of this paper; QED.

NOMENCLATURE

A = amplitude of gravitational wave (GW) variation with time (m/m)
c = speed of light, $2.998\text{x}10^8$ (ms^{-1})
D = distance from GW focus (m)
f = force (N)
$\boldsymbol{f}_{cf}$ = centrifugal-force vector (N)
F = GW flux (Wm^{-2})
G = universal gravitational constant = $6.693\text{x}10^{-11}$ ($\text{m}^3/\text{kg-s}^2$),
h = metric perturbation, the GW spacetime strain as a function of time (m/m)
h_{ij} = $\delta h_{ij}/\delta t$
$\dot{h}_{ij} = \dfrac{\partial h_{ij}}{\partial t}$
I = moment of inertia (kg-m^2)
P = magnitude of the power of a gravitational-radiation source (W)
r = radial distance to an object; alternately, the effective radius of gyration, or r = $(x^2 + y^2)^{1/2}$(m)
t = time (s)
Δ = small increment
Δf_{cf} = increment of centrifugal force change (N)
Δf_t = increment of tangential force change (N)
Δf_i = individual FBAR force change (N)
Δt = time increment (s)
λ = wavelength (m)
ν = frequency (s^{-1})
ω = angular rotational rate (rad/s)
n = number of FBAR elements in phase

Subscripts

cf centrifugal
EM electromagnetic
GB Gaussian beam
GW gravitational wave
i individual
t tangential
x component along the x-axis

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- **RESULT 1: The HFGW amplitude for the laser and for the Magnetron-FBAR gravitational-wave generator is proportional to the slope of the force versus time curve and this provides a template amplitude verses time for HFGW detectors.**
-
- **RESULT 2: There is a design-parameter relationship or "figure of merit" for a high-frequency gravitational wave laboratory generator comprising a number of vibrating masses or elements (e.g., piezoelectric crystals or FBAR pairs), which are lined up and in phase, that states:**
- **The amplitude of the generated gravitational radiation is proportional to:**

  The distance between the individual vibrating masses (e.g., the width of the in- line, in-phase piezoelectric crystals or the distance between in-line, in-phase oppositely directed FBAR pairs).

  The change in force of the vibrating masses during each cycle

  The frequency of the generated gravitational radiation and

  The square of the number of in-line, in-phase vibrating masses or elements.

- **RESULT 3: Utilizing the approximate engineering or jerk approach to high-frequency gravitational wave (HFGW) power estimation, it appears that the change in force each cycle, which occurs in the piezoelectric crystals used by Dehnen and Romero-Borja, is 8.704 milli-newtons for the 3 GHz case, 0.4201 milli-newtons for the 1300 GHz case and when corrected for the frequency is within half of a percent of each other thus confirming the jerk approach.**

- **CONCLUSION 1: The approximate engineering or jerk approach to estimating the power of laboratory-generated high-frequency gravitational waves provides reasonable results to within one-half percent when compared with the far more elaborate and rigorous Dehnen and Romero-Borja General Relativity approach.**

- **CONCLUSION 2: If one can detect high-frequency gravitational waves in the GHz frequency range having amplitudes of about 10-24 to 10-29 δm/m (less sensitivity than required for HFRGW detection), then a laboratory generation/detection experiment is possible utilizing off-the-shelf components.**

- **CONCLUSION 3: Future embodiments utilizing nanotechnology could reduce the array size to cm's.**

Informal Comparative Analysis of Dehnen and Romero-Borja's HFGW-Generation Calculations with those of Baker, Stephenson and Li

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Abstract. A comparative analysis is presented for the engineering or jerk approach to the estimation of the high-frequency gravitational wave (HFGW) power utilized by Baker (2006) to the more rigorous, general-relativity approach utilized by Dehnen and Romero-Borja (2006; 1981). The values for the power of the laboratory generated HFGWs by Dehnen and Romero-Borja at two different HFGW frequencies, 3 GHz and 1300 GHz, leads to changes in the force in the piezoelectric crystals computed by employing the jerk-approach equations. These force changes or jerks are computed to be 8.704 milli-newtons and 0.4201 milli-newtons, respectively. These forces are quite reasonable for the 1981-vintage crystals compared with 2008 milli-newtons for the modern, new-technology based crystals utilized in cell-phone film bulk acoustic resonators (FBARs). In fact, the results are within one-half of one percent of each other when corrected for the square root of the frequency. It is concluded that this theoretical result confirms the validity of the jerk approach to HFGW power estimation already checked against a similar calculation for PSR1913+16 given in Baker (2006).

Keywords: Microwaves, General Relativity**,** High-Frequency Gravitational Waves, Piezoelectric.
PACS: 04.30.Db, 04.80.Nn, 84.40.Fe, and 77.65.-j.

INTRODUCTION

This comparative analysis is of the "engineering" or simplified approach of the paper delivered at HFGW2 Workshop (Baker, Stephenson and Li, 2007) with another more rigorous approach to the laboratory generation of HFGWs based on the theory of General Relativity (Dehnen and Romero-Borja, 2003;1981). It is found that the results of the two approaches provide results that are reasonably consistent. That being the case, the conclusion of the HFGW2 Workshop presentation on Thursday, September 20th, 2007 that it may be possible now to generate such detectable GW radiation in the laboratory using off-the-shelf components (e.g., microwave Magnetrons and cell-phone FBAR piezoelectric crystals), might be correct.

With regard to General Relativity (GR), it is assumed that the papers by Dehnen and Romero-Borja (2003;1981) and the Appendix by Fangyu Li in Baker, Stephenson and Li (2007) concerning the relationship of gravitational wave (GW) amplitude to GW flux and frequency are correct. The approach is to adopt the Dehnen and Romero-Borja design of HFGW laboratory generation and scale it to the parameters and range of parameters of the Magnetron-energized FBAR approach of Baker, Stephenson and Li (2007) using the parameters for the piezoelectric crystal (including FBARs) elements provided by Woods and Baker (2005). Specifically, the jerk-approach equations derived in Baker (2006) are employed to estimate the change in force or jerk in the crystals utilized in the two frequency cases (3 GHz and 1300 GHz) considered by Dehnen and Romero-Borja (2003). These forces are corrected as to frequency and compared in order to validate the Baker, Stephenson and Li approach.

GW POWER

The equation for the GW power, P, in terms of the time-rate-of-change of acceleration or jerk is derived in Baker (2006). . The equation is of two forms, with $2r$ the distance between two jerking masses (e.g., twice the distance between the rows or tracks of FBARs and the centerline where GWs are generated) or the linear dimension of a single crystal's diatomic linear chain, i.e., the thickness of a single piezoelectric crystal (m), Δf the total force change (N), Δt (s) the time interval of the force change and ν is the energizing frequency (s^{-1}):

$$P(r, \Delta f, \Delta t) = 1.76\text{x}10^{-52} (2r\Delta f/ \Delta t)^{2} \text{ W} \quad (1)$$

and

$$P(r, \Delta f, \nu) = 1.76\text{x}10^{-52} (2r\nu\Delta f)^{2} \text{ W}. \quad (2)$$

The GW flux or $F_{GW} = P/\delta A$ in which the reference area δA is equal to some factor, k, of the area of the diffraction pattern having diameter (to the first GW diffraction ring) d. We set d = 1.2 λ_{GW} in which λ_{GW} is the GW wave length = c/ν_{GW}, c being the speed of light and ν_{GW} is the frequency of the GW or twice the energizing frequency, ν. So that $\delta A = k\pi d^2/4 = k\ \pi c^2/4\nu_{GW}^2$. Thus when we include the number of elements N in the HFGW generator's linear array the GW flux is (with $\nu_{GW} = 2\nu$):

$$F_{GW} = 1.76\text{x}10^{-52}(2r\nu\ N^2\Delta f_i)^{2}/ [(k\ \pi c^2/4\nu_{GW}^2)/N^2] \text{ proportional to } r^2\nu^4\ \Delta f_i^2\ N^4 \quad \text{Wm}^{-2}. \quad (3a)$$

Alternatively, one can utilize the gravitational-wave radiation pattern derived by Landau and Lifshitz (1975, 4th English edition, pp. 355-356) and the flux equation derived from it in Baker, Davis and Woods (2005):

$$F_{GW}\pm10^0 = P_0\ 2.54\ (0.282/D)^{2} \text{ W m}^{-2}, \quad (3b)$$

where $F_{GW}\pm10^0$ is the HFGW flux in a 10^0 cap of the radiation pattern in the direction of the propagating HFGWs (and assumed to be in the direction of the build up of the coherent HFGWs) and D is the distance from the end of the train or parallel strings of coherent FBARs (or piezoelectric crystals) expressed in terms of the number of gravitational wavelengths, n (i.e., $D = n\lambda_{GW} = nc/\nu$) or:

$$F_{GW}\pm10^0 = 2.54\text{x}1.\ 76\text{x}10^{-52}(2r\nu\ N^2\Delta f_i)^{2}\ (0.282\nu_{GW}/nc)^{2} \text{ W m}^{-2}, \quad (3c)$$

where n must be greater than one in order to avoid diffraction, e.g., n = 1, 1.5, 2, etc.

GW AMPLITUDE

Equation (4) of Baker, Stephenson and Li (2007) is utilized to compute the amplitude A of the laboratory piezoelectric-generated HFGWs:

$$A = 1.28\text{x}10^{-18}\ F_{GW}^{1/2}/\ \nu_{GW} \quad \delta\text{m/m}. \quad (4)$$

Thus, in summary, the design-parameter relationship or "figure of merit" for a HFGW laboratory piezoelectric crystal generator is:

$$A \text{ is proportional to } r\nu_{GW}\ \Delta f_i\ N^2. \quad (5)$$

This also the result given in Baker, Stephenson and Li (2007).

To compute the amplitude of the laboratory generated HFGW using the results of Dehnen and Romero-Borja (2003), we will utilize their first example given by Eqs. (4.50) and (4.51) since its frequency range is closest to that of the Magnetron-FBAR generator. In this case the frequency is $\nu = 3\text{x}10^9$ Hz and flux of $F_{GW} = 1.7\text{x}10^{-20}$ Wm^{-2} . Thus:

$$A = 1.28\text{x}10^{-18}\ F_{GW}{}^{1/2}/(\ \nu_{\text{GW}})\ \ \delta\text{m/m} = 1.8\text{x}10^{-37}\ \ \delta\text{m/m}\ . \qquad (6)$$

In Dehnen and Romero-Borja (2003) it is stipulated that the distance between the "masses" of the vibrator or ends of the diatomic linear chain of a single crystal is *b*, whereas the distance between crystals, *a* must be

$$a << \lambda_{\text{GW}} \qquad (7)$$

where λ_{GW} is the HFGW wavelength for the frequency of the HFGW. In the examples of Dehnen and Romero-Borja the *a* is taken as the thickness of their piezoelectric crystals or 10^{-5} m (please see their Fig. 5) whereas Baker, Stephenson and Li adopt a *r* (half the distance between the masses or FBAR pairs) of from about a tenth of a wavelength (0.0061 m) to one kilometer. Here we are on uncertain ground, but the requirement that 2 *r* or 2*b* or $a << \lambda_{GW}$ may ***not*** be a stringent or even a necessary one for the quadrupole approximation to GW power to hold. As K. S. Thorne (1987) states "… the quadrupole formalism typically is accurate to within factors of order 2 even for sources with sizes of (the) order (of) a reduced (GW) wavelength …" Whether the quadrupole approximation to the power of gravitational wave generation holds accurately or not does not necessarily imply that no GWs are generated by an impulsive force acting on a pair of masses or that the power does not increase with the distance, *2r* (or *2b* for Dehnen and Romero-Borja) between the radiating masses equal to or greater than a GW wavelength. The quadrupole formalism may still provide order-of-magnitude estimates perhaps augmented by higher-order octupole, hexadecapole, etc. modes of pulsation or jerk and possibly reduced at the GW focus by diffraction. Also the output power of the HFGW cannot exceed the power of the energizing Magnetrons. It is a problem deserving study in future.

CHANGE IN FORCE OR JERK

The piezoelectric crystals considered by Dehnen and Romero-Borja, 2003 are of a 1981 vintage and far less efficient than the modern FBARs that are a product of new technology, especially advanced cell-phone designs. From Eq. (8) of Woods and Baker (2005), the change in force of an individual element (e.g., FBAR) is given by

$$\Delta f_i = (2QP_i\omega_0 m)^{1/2}\ \text{N}, \qquad (8)$$

where Q is the resonance quality factor, P_i is the power absorbed by the individual FBARs (for a 1 Kw Magnetron distributing its energy among three FBAR wafers having 6000 elements each, the P_i = 1000/3x6000 = 56 mW, which is well below the 2 W power capacity reported by Ruby et al., 1999), ω_0 is the natural angular frequency = $2\pi\nu$ and *m* is the mass of the vibrating element (100x100x1μm^3 x 3000 kgm^3 = $3\text{x}10^{-11}$ kg or 30 ng for an FBAR and 10 grams or 10^{10} ng for the much larger and less modern Dehnen and Romero-Borja crystals). A typical FBAR has a resonance curve with a pass-band resonance width of $2\Delta\nu = 24$ MHz at a typical pass-band center frequency $\nu_0 = 2$ GHz (Lakin et al., 2001). This gives a Q = 2000/24 = 83. For the Baker, Stephenson and Li Magnetron-FBAR system with $\nu = 2.45\text{x}10^9$, Eq. (8) yields:

$$\Delta f_i = (2\text{x}83\text{x}0.056\text{x}6.28\text{x}2.45\text{x}10^9\text{x}3\text{x}10^{-11})^{1/2} = 2.08\ \text{N}. \qquad (9)$$

For the Dehnen and Romero-Borja case the use of Eq. (2) for *r* (or in their case *b*) or *a* = 0.00001 m for the two cases (1) $\nu_{GW} = 3$ GHz and *P* = 0.48 attowatts and (2) $\nu = 1300$ GHz and *P* = 210 attowatts, yields Δf_i = 8.704 mN and 0.4201 mN, respectively. It is difficult to compare with Baker, Stephenson and Li since Dehnen and Romero-Borja do not consider the energizing power, but the square root of frequency difference would increase the 0.4201 N Δf_i to $0.4201\text{x}(1300/3)^{1/2}$ = 8.745 mN, which is only a factor of 8.745/8.704 = 1.0047 or about half of a percent different. Thus the simplified, engineering Baker, Stephenson and Li approach gives results that are quite close to the more complete GR approach of Dehnen and Romero-Borja even over quite different frequency ranges. Evidently, from Eq. (8)

$$QP_i = \Delta f_i^2/\omega_0 m = 2\text{x}10^{-13}\ \text{N} \tag{10}$$

for the Dehnen and Romero-Borja piezoelectric crystals and their other results follow directly from the jerk equations, i.e., Eq. (2).

By the way, the length of the Dehnen and Romero-Borja oscillator row is $Na = 10^7\text{x}10^{-5} = 100$ m, whereas the two tracks or parallel rows of the FBARs (assuming that their square faces are "face up" on one track and "face down" on the other track for oppositely directed Δf_i) are 110 μmx1.8x10^8 = 19.8 km long. Please see Fig. 1 for a depiction of the design. This is a rather long array, but as Dr. Hal Puthoff (E-mail dated October 2, 2007) noted: "…just as for ELF communications to submarines (Project Sanguine, then Project Seafarer - see http://en.wikipedia.org/wiki/Communication_with_submarines) such large antenna arrays are standard fare for such communication (systems)." The length could be greatly reduced if each track consisted of several close (110μm) parallel, staggered rows of FBARs as shown schematically in Fig. 2. Such a design would also allow for the power of the Magnetron beam (probably focused) to be more completely absorbed by the FBARs. The power requirement for the 20,000 Magnetrons on the two tracks would be at least 20 MW, so that a power substation of that size would be required.

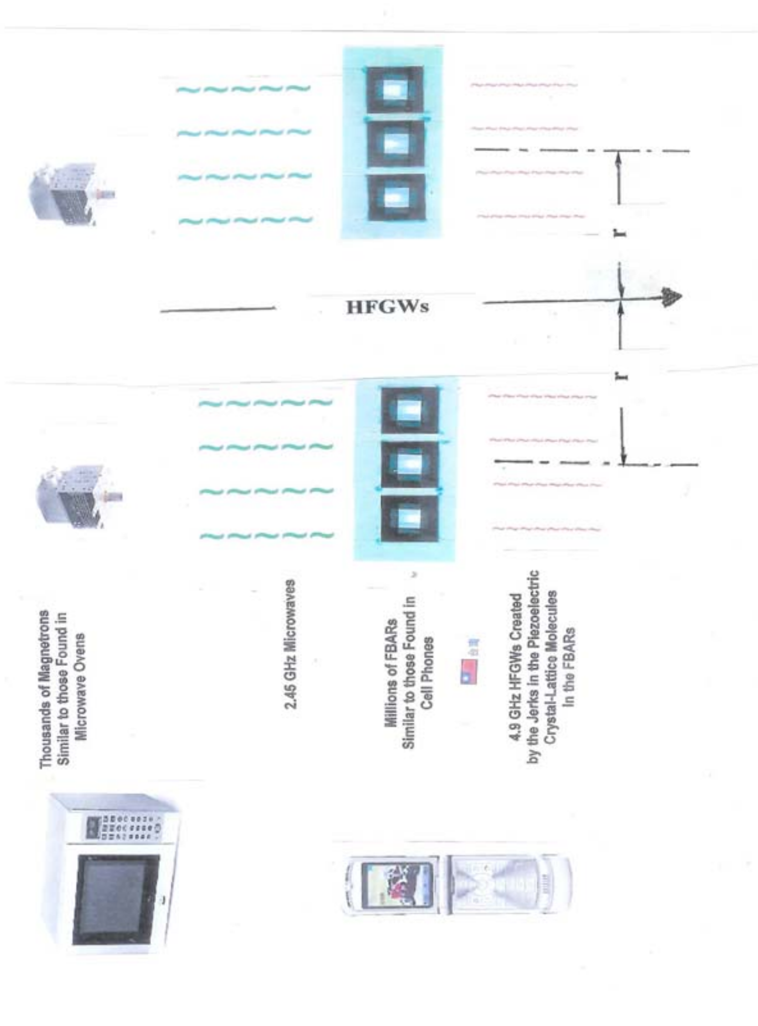

FIGURE 1. Depiction of the Magnetron-FBAR HFGW Generator Design

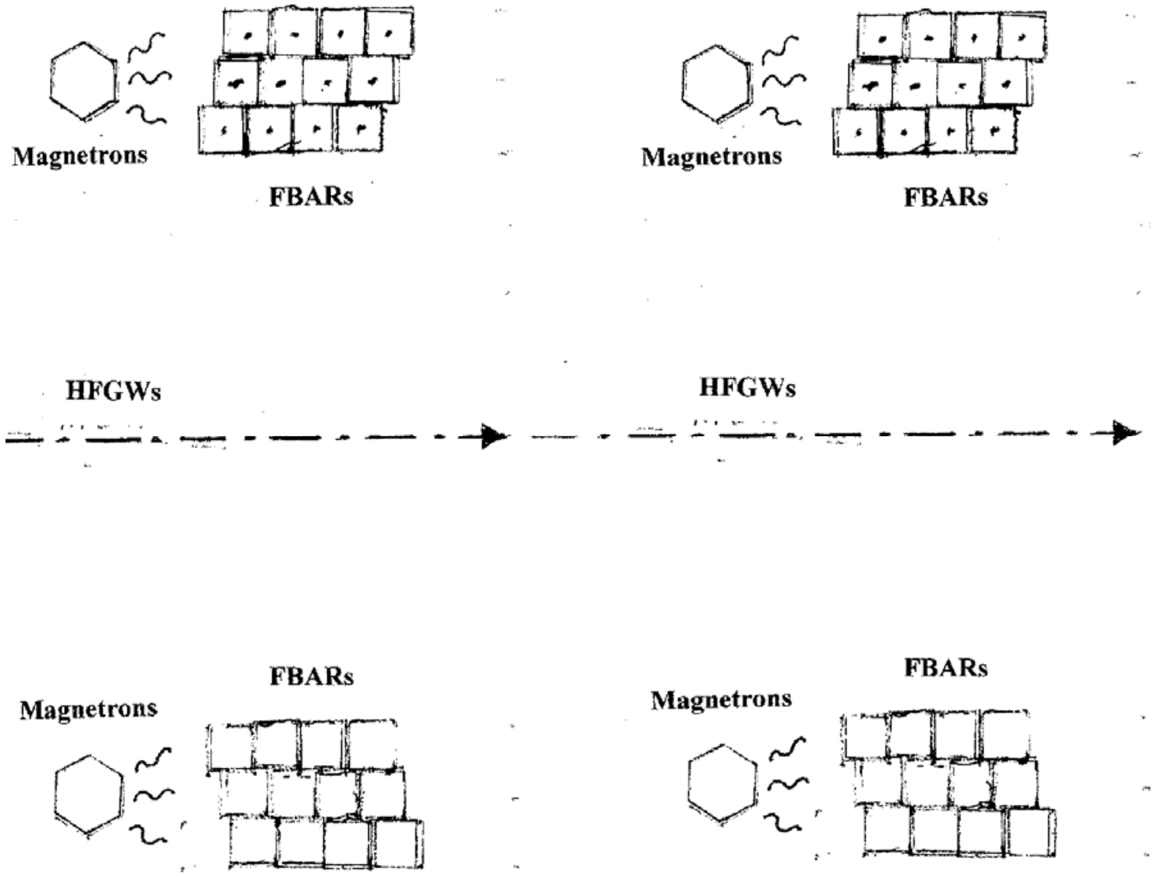

FIGURE 2. Illustration of the Parallel Staggered Tracks of FBARs

The power, flux and HFGW amplitude for the Magnetron-FBAR generator will be determined for the cases of $r = 0.0061$m (one tenth of a HFGW wave length at 4.9 GHz), 0.0305 (one half of a wavelength), 0.061 m (one wavelength), 300 m and 1 km at a detector distance of 1.5 HFGW wavelengths from the end of the FBAR array. The results are given in the following Table 1:

TABLE 1. HFGW Amplitude *A* for Various Separation Distances *r* of the FBAR Pairs from the Centerline.

r (meters)	Power (watts)	Flux (watts per square meter)	A (δm/m)
0.0061	7.14×10^{-4}	1.713×10^{-2}	3.42×10^{-29}
0.0305	1.78×10^{-2}	0.428	1.71×10^{-28}
0.061	7.14×10^{-2}	1.713	3.42×10^{-28}
300	1.73×10^{6}	4.14×10^{7}	1.68×10^{-24}
1000	1.92×10^{7}	4.60×10^{8}	5.6×10^{-24}

It is conceivable that *N* could be increased by a factor of ten, in which case *A* would be on the order of 10^{-26} to 10^{-22} (m/m).

CONCLUSIONS

The approximate engineering or jerk approach to estimating the power of laboratory generated high-frequency gravitational waves provides reasonable results when compared with the far more elaborate and rigorous Dehnen and Romero-Borja approach. If one can detect high-frequency gravitational waves in the GHz frequency range having amplitudes of about 10^{-24} to 10^{-29} (meters per meter), which the Chinese detector is sensitive to (Li and Baker, 2007), then a laboratory generation/detection experiment is possible utilizing off-the-shelf components.

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Generation of GHz - THz Band, High-Frequency Gravitational Waves in the Laboratory

(Paper HFGW-03-102)

by
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ABSTRACT

The generation of high-frequency gravitational waves (HFGW) by a coherent excitation of a 100 [m] long row of oscillators is investigated. As oscillators we choose ultra-thin, $1x10^{-5}$ [m] (or 0.01 [mm]) thick piezoelectric crystals, which are described as an idealization by diatomic, 0.6 [m] (60 [cm]) long linear chains. We find a highly focused super radiant beam of gravitational radiation in direction of the row (needle radiation beam pattern) and a total radiation power larger than the incoherent superposition of the oscillator radiation by the factor λ/a (where λ is the wavelength of the HFGW and a is the distance between neighboring oscillators). It appears that under optimum conditions the attainable radiation power of a row of $5x10^7$ oscillators is approximately on the order of $5x10^{-19}$ to $2x10^{-16}$ [watts]. Whether or not this order of magnitude radiated power can be enhanced by a high-temperature superconductor lens to a flux of $1.3x10^{-9}$ [watts/m^2] for 1.3 THz HFGW and whether or not this flux can be detected by modern observational techniques, to be described by other conference papers, would be an outcome of the experiment to be developed at this Conference.

1. INTRODUCTION

From the beginning of the "gravitational-radiation era," based primarily on the pioneering work of Weber [1], one was almost exclusively dedicated to develop appropriate antennae for the *detection* of the theoretically predicted gravitational waves coming from celestial bodies. We can not say, certainly until now, that a *direct* detection of this radiation in the laboratory succeeded. On the other hand, only a few authors have given estimations about the possibility of laboratory experiments for the *generation* and subsequent detection of gravitational

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 The basis of this paper is a publication of F. Romero and H. Dehnen (1981), *Z.Naturforsch*, **36a**, 948.

waves. However these results have not found a consensus until now. For this reason we intend to show in this paper by means of a particular arrangement of high frequency oscillators that the generation in the laboratory is really a hard task even in case of most favourable conditions. In comparison with other authors we do not assume any special excitation mechanism for our arrangement, leaving this decision to the experimentators; we just demand an optimum on-phase coherent excitation which should give a highly focussed superradiant beam of gravitational radiation and a kind of stimulated emission.

We perform the whole calculation within the linear theory of gravitation and give at first a brief summary of the most important results used in the work. Assuming for the space-time metric the weak-field approximation ($\eta^{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$):

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}, \quad |h^{\mu\nu}| \ll 1, \tag{1.1}$$

with the gauge condition

$$\hat{h}^{\mu\nu}{}_{|\nu} = 0, \quad \hat{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2} h \eta^{\mu\nu}, \quad (h \equiv h^{\mu\nu} \eta_{\mu\nu}) \tag{1.2}$$

the field equations read ($G = 1, c = 1$)

$$\Box \hat{h}^{\mu\nu} = -16\pi T^{\mu\nu}. \tag{1.3}$$

The well known far-field solution of (1.3) (see for example [2]) is

$$\hat{h}^{\mu\nu}(t, \mathbf{x}) = \frac{4}{r} \int T^{\mu\nu}(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}') d^3x', \tag{1.4}$$

whose space-like components[1] can be set within the "quadrupole formalism" into the form

$$\hat{h}^{jk}(t, \mathbf{x}) = \frac{2}{r} \frac{d^2}{dt^2} \int \rho(t', \mathbf{x}') x'^j x'^k d^3x', \tag{1.5}$$

where $t' \simeq t - r$ is the retarded time between field point and center of mass of the source (ρ mass-density). This quadrupole approximation is only valid for the low-frequency limit $\lambda_{GW} \gg a$, a linear dimension of the source, which implies slow-motions within the source.

For the energy flux of the radiation in radial direction we have [3]

$$T^{0r}_{GW} = \frac{1}{32\pi} \langle \hat{h}^{TT}_{jk|0} \hat{h}^{TT}_{jk|0} \rangle \tag{1.6}$$

(bracket means average over several wave lengths), wherein

$$\hat{h}^{TT}_{jk} = P_{jl} P_{mk} \hat{h}_{lm} - \frac{1}{2} P_{jk} (P_{ml} \hat{h}_{lm}) \tag{1.7}$$

[1]Latin indices run from 1 to 3, greek indices from 0 to 3. Sum convention is used.

represents the transverse and traceless projection of the metric perturbation performed by the projection tensor $P_{jk} = \delta_{jk} - n_j n_k$, which projects on the 2–dimensional plane orthogonal to the propagation direction of the wave $n_k = k_k/|\mathbf{k}|$ (k_k 3–dimensional wave vector). In this projection $\hat{h}^{TT}_{jk}$ coincides with h^{TT}_{jk}.

2. THE MODEL OF THE SOURCE AND ITS RADIATION FIELD

We describe in the following a source for gravitational radiation whose excitation allows a type of "superradiance". Such a model is represented by a row of identical oscillators (two–mass vibrators), which are ordered on a line with constant distance with respect to one another. The vibrators have their axes mutually parallel and are located orthogonal to the row axis, which is chosen identical with the y–axis. The arrangement and the allowed displacements of the single vibrators are shown in Figs. 1 and 2, respectively, whereby the single vibrator has the following characteristics: two equal masses M coupled harmonically by a spring and separated by a distance $2b$ at the equilibrium position. The distance between two neighbouring vibrators is a and the row begins at $y = 0$ and finishes at $y = (N-1)a$, where N is the total number of vibrators in the row.

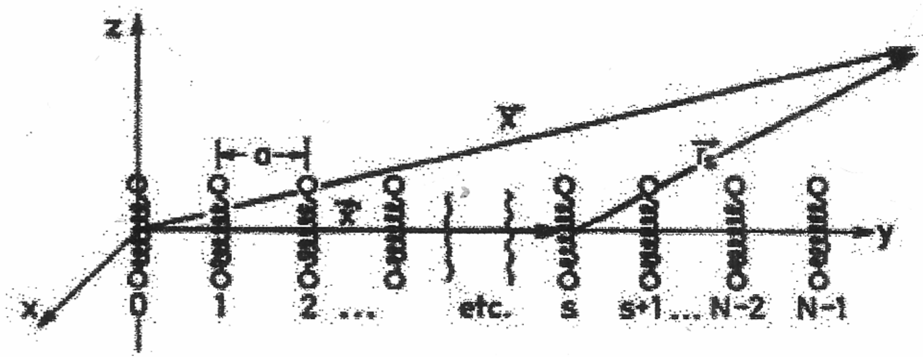

Figure 1: The whole row of vibrators.

Next we calculate the gravitational radiation of this device, whereby an adequate on–phase excitation of the elements is assumed in order to obtain a highly focussed radiation of the antenna and a certain superradiance in the beam. The displacements out of the equilibrium position for the masses of the

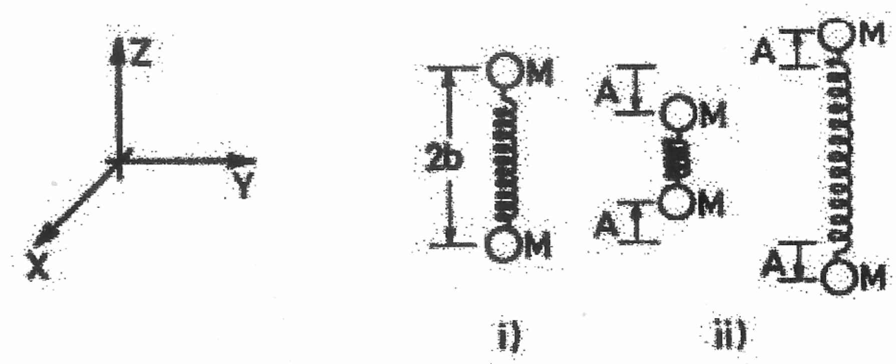

Figure 2: The single vibrator of the row in three positions. i) Equilibrium position ii) possible displacements.

s-th vibrator take the form

$$u^s(t') = \pm(b + A\sin[\Omega(t' + \Phi(s))]) \tag{2.8}$$

(Ω frequency, A amplitude and Φ phase). The phase $\Phi(s)$ will be determined later in such a way that a positive superpcsition of the emitted radiation in the direction of the y–axis results. Then the mass distribution along the row reads

$$\rho = M\sum_{s=0}^{N-1}\{\delta^3(\mathbf{x}' - [sa\mathbf{e}_y + u^s(t')\mathbf{e}_z]) + \delta^3(\mathbf{x}' - [sa\mathbf{e}_y - u^s(t')\mathbf{e}_z])\} \tag{2.9}$$

with $\mathbf{e}_y$ and $\mathbf{e}_z$ unit vectors in direction of the $y-$ and z–axis, respectively, wherein we have considered that the vibrators are located parallel to the z–axis with their centers of mass on the y–axis. Since the single vibrator can be considered as a closed system (${T_\mu}^\nu{}_{|\nu} = 0$), which interacts with the neighbouring system only through the freely disposable phase Φ, it is justified to use the quadrupole formalism mentioned above for each single vibrator and to superpose subsequently the wave amplitudes produced by all vibrators of the row in a coherent way (for an alternative procedure of obtaining the total radiation field see for example [4]).

According to (1.7) and (1.5) the radiation field of the s-th vibrator reads

$$\overset{sTT}{h_{jk}} = \frac{2}{r_s}\ddot{Q}^{TT}_{jk}(t - r_s + \Phi(s)), \tag{2.10}$$

where

$$Q_{jk} = I_{jk} - \frac{1}{3}\delta_{jk}I, \quad I = I_{jk}\delta_{jk}, \quad I_{jk} \equiv \int \overset{s}{\rho}(\mathbf{x}', t')x'_j x'_k d^3x' \tag{2.11}$$

is the mass–quadrupole tensor and r_s the distance from the center of mass of the s–th vibrator to the field point. The mass distribution for the single vibrators is given by the single terms of (2.9) as

$$\overset{s}{\rho} = M\{\delta^3(\mathbf{x}' - [sa\mathbf{e}_y + u^s(t')\mathbf{e}_z]) + \delta^3(\mathbf{x}' - [sa\mathbf{e}_y - u^s(t')\mathbf{e}_z])\}. \qquad (2.12)$$

For the whole vibrator row the radiation field results in

$$h_{jk}^{TT} = \sum_{s=0}^{N-1} \overset{s}{h}{}_{jk}^{TT} . \qquad (2.13)$$

Inserting (2.10) into (2.13) we get

$$h_{jk}^{TT} = \frac{2}{r} \sum_{s=0}^{N-1} \ddot{I}_{jk}^{TT}(t - r_s + \Phi(s)), \qquad (2.14)$$

where $r_s \simeq r - sa\sin\theta\sin\phi\,(\theta, \phi$ usual polar angles). The demand for constructive superposition of the radiation of the row in direction of the y–axis requires for the phase–shift $\Phi(s) = -sa$, which may be realized by a computer controlled logic system. Herewith and with (2.12) one obtains from (2.11) for small vibration amplitudes ($A \ll 2b$) after projection according to (1.7)

$$\ddot{I}_{jk}^{TT} = -2MAb\Omega^2 \cdot \sin[\Omega(t-r) + sa(\sin\theta\sin\phi - 1))] \cdot (1 - n_z^2)e_{jk}. \quad (2.15)$$

Insertion into (2.14) gives

$$h_{jk}^{TT} = \frac{-4MbA\Omega^2}{r}(1-n_z^2) \sum_{s=0}^{N-1} \{\sin[\Omega(t - r + sa(\sin\theta\sin\phi - 1))]\}e_{jk} \quad (2.16)$$

for the total radiation field of the row, where e_{jk} is the polarization tensor defined by

$$e_{jk} = \frac{2}{\sin^2\theta}\{(\delta_{jz} - n_j\cos\theta)(\delta_{kz} - n_k\cos\theta) - \frac{1}{2}\sin^2\theta(\delta_{jk} - n_j n_k)\} \quad (2.17)$$

with the following general properties:

$$e_{jk}n^k = 0, \quad e_j{}^j = 0, \quad e_{jk}e^{jk} = 2. \qquad (2.18)$$

For the beam in direction of the y–axis it takes the simple form

$$e_{jk} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \qquad (2.19)$$

Summing up the terms in (2.16) we get

$$h_{jk}^{TT} = \frac{-4MbA\Omega^2}{r}(1-n_z^2)\frac{\sin\left[N\frac{\Omega a}{2}(n_y - 1)\right]}{\sin\left[\frac{\Omega a}{2}(n_y-1)\right]} \qquad (2.20)$$

$$\cdot\{\sin[\Omega(t - r + (N-1)(a/2)(n_y - 1))]\}e_{jk}.$$

Herewith we obtain from (1.6) for the energy flux in radial direction:

$$T^{0r}_{GW} = \frac{1}{2\pi}\frac{(MbA)^2}{r^2}\Omega^6 \frac{\sin^2\left[N\frac{\Omega a}{2}(n_y-1)\right]}{\sin^2\left[\frac{\Omega a}{2}(n_y-1)\right]}(1-n_z^2)^2. \tag{2.21}$$

This expression has the following properties of interest:

a) In case that $N = 1$ (the row is reduced to a single vibrator) it becomes independent of ϕ; the resulting expression agrees exactly with the energy flux for a vibrator (compare for example [5]) with its well–known (ϕ independent) angular distribution

$$T^{0r}_{GW} = \frac{1}{2\pi}\frac{(MbA)^2}{r^2}\Omega^6 \sin^4\theta \tag{2.22}$$

and the total radiation power

$$L_{GW} = \int T^{0r}_{GW} r^2 \sin\theta d\theta d\phi = \frac{16}{15}(MbA)^2\Omega^6 \hat{=} \frac{16}{15}\frac{G}{c^5}(MbA)^2\Omega^6. \tag{2.23}$$

b) In case of several vibrators the expression (2.21) shows a strong direction dependence for the emission of the generated gravitational waves; for $\lambda_{GW} \gg 2a$ (only one zero–point of the denominator in (2.21)) we find a very strong emission in direction of the y–axis and a minimum or vanishing emission in all other directions including the negative y–direction (see Fig. 3). Consequently we have a typical focusing of the radiation, which possesses additionally a superradiant behaviour (radiation intensity $\sim N^2$); for the beam in direction of the y–axis it follows:

$$T^{0y}_{GW} = \frac{1}{2\pi}\frac{(MbA)^2}{r^2}\Omega^6 N^2 \hat{=} \frac{1}{2\pi}\frac{G}{c^5}\frac{(MbA)^2}{r^2}\Omega^6 N^2. \tag{2.24}$$

In view of this superradiance the possibility of a laboratory experiment should be investigated. First, however, a more realistic description of the device is necessary. This and an estimation of the obtainable radiation power are given in the two next sections.

Finally we analyse the properties of the total radiation power obtained by integration of (2.21) over the total sphere; for this we confine ourselves to the case $N \gg 1$ and $\lambda_{GW} \gg 2a$ (maximum focusing). Then the ratio of the sin–functions in (2.21) possesses a very sharp maximum only for $\theta = \phi = \pi/2$, given by (2.24); this high intensity exists only within the very small angles (half–width angles)

$$\left.\begin{matrix}\Delta\theta \\ \Delta\phi\end{matrix}\right\} = \pm 2\sqrt[4]{2}(\Omega a N)^{-1/2} \tag{2.25}$$

around $\theta = \phi = \pi/2$ ("needle radiation"). Accordingly the integration of (2.21) can be restricted to this angular region. So we find for the total radiation power (see appendix):

$$L_{GW} = 2.8(MbA)^2\Omega^5 N/a \hat{=} 2.8G(\Omega^5/c^4)(MbA)^2 N/a. \tag{2.26}$$

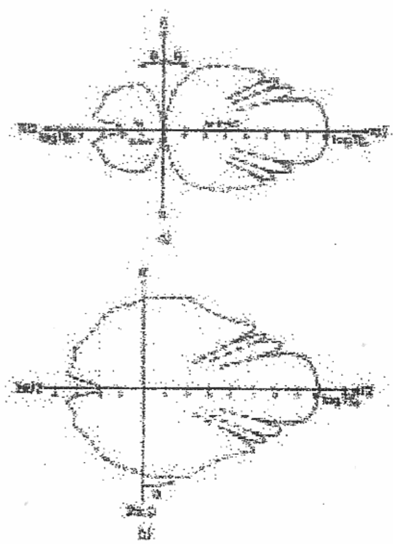

Figure 3: Angular distribution for the gravitational radiation of the vibrator–row (formula (2.21) with $N = 10^4, \Omega = 10^9$ sec^{-1} and $a = 0.5$ cm; the values for $\log T^{0r}_{GW}$ correspond alone to the angular part of T^{0r}_{GW}): a) $\phi = \pi/2, 3\pi/2; \theta$ runs b) $\theta = \pi/2, \phi$ runs. Evidently there exists a "needle radiation" in direction of the y–axis ($\theta = \phi = \pi/2$). This becomes more pronounced for increasing values of the product $\Omega a N$, see (2.25).

This result shows that because of the linear dependence of the right hand side of (2.26) on N no superradiance for the total power seems to exist. On the other hand we have an astonishing dependence on frequency and light velocity of the form Ω^5/c^4, which is unusual for gravitation. This peculiar dependence is responsible for the fact that the radiation power (2.26) is larger than the N–fold power of the single vibrator: Comparing (2.23) and (2.26) we obtain

$$\frac{L_{row}}{N L_{vibrator}} = 2.8(15/16)\lambda_{GW}/a \gg 1. \qquad (2.27)$$

This means that a remnant of superradiance or a kind of stimulated emission is present.

3. GRAVITATIONAL RADIATION OF THE DIATOMIC FREE VIBRATING FINITE LINEAR CHAIN

Since the calculation given above with the two–mass vibrators represents only a strong idealization of a laboratory experiment, the necessity exists to give a more realistic description and analysis for the single vibrator. In case of an expriment the vibrator would be realized practically by a thin piezoelectrical crystal; in this sense we consider in the following a diatomic linear chain as a useful model for such a solid and compare subsequently its radiation power with that of the vibrator. So the vibrator data can be fitted in such a way that the single vibrator represents appropriately a thin piezoelectrical crystal.

The total number of atoms of the chain may be $2N'$; this means we have N' atoms with mass M_1 and N' atoms with mass M_2, which lay alternately on a straight line (z–axis) separated by a distance a', so that the lattice parameter is $l = 2a'$. Further we consider an harmonic interaction between next neighbours only described by the spring constant β. The Fig. 4 shows the chosen arrangement.

The equations of motion for the two sorts of masses are:

$$\begin{aligned} M_1\ddot{u}_1^s &= -\beta(2u_1^s - u_2^s - u_2^{s+1}), \\ M_2\ddot{u}_2^s &= -\beta(2u_2^s - u_1^s - u_1^{s-1}), \end{aligned} \tag{3.28}$$

where we have confined ourselves to the longitudinal vibrations u_j^s in view of a reasonable comparison between chain and vibrator. The eigensolutions of (3.28) for the *free*–vibrating *finite* chain are treated in detail in [4]; they take the following normalized form:

Figure 4: The diatomic linear chain at the equilibrium position.

$$\begin{aligned} u_{1m_i}^s(t) &= B_{m_i}D_{m_i}A_1\cos[2s\kappa_i a' - \Psi_1(m_i)]\cos[\Omega_{m_i}t + \gamma_{m_i}], \\ u_{2m_i}^s(t) &= B_{m_i}D_{m_i}A_2\cos[2s\kappa_i a' - \Psi_2(m_i)]\cos[\Omega_{m_i}t + \gamma_{m_i}] \end{aligned} \tag{3.29}$$

with

$$D_{m_i} = \sqrt{\frac{2(M_1+M_2)}{(M_1A_1^2+M_2A_2^2)}} \tag{3.30}$$

and

$$\begin{aligned} \Psi_1(m_i) &= \text{arctg}\left[\frac{(A_1/A_2)_{m_i} - \cos\kappa_i a'}{\sin\kappa_i a'}\right], \\ \Psi_2(m_i) &\equiv \kappa_i a' + \Psi_1(m_i). \end{aligned} \tag{3.31}$$

The subindex m_i is the mode index ($m_i = 0, 1, 2, ..., (N'-1)$), wherein i stands for the acoustical branch ($i = 1, -$sign) and the optical branch ($i = 2, +$sign)of the well-known dispersion relation:

$$\Omega_{m_i}^2 = \frac{\beta}{M_1M_2}\{M_1 + M_2 \pm [M_1^2 + M_2^2 + 2M_1M_2\cos(2\kappa_i a')]^{1/2}\}. \tag{3.32}$$

For the ratio of the amplitudes A_1, A_2 in (3.29) one finds:

$$(A_1/A_2)_{m_i} = \frac{(2\beta/M_1)\cos(\kappa_i a')}{(2\beta/M_1) - \Omega_{m_i}^2} = \frac{(2\beta/M_2) - \Omega_{m_i}^2}{(2\beta/M_2)\cos(\kappa_i a')}, \tag{3.33}$$

whereas the values of B_{m_i} and γ_{m_i} are determined by the initial conditions. From the free boundary condition, that means $u_1^0 = u_2^1$ and $u_1^{N'} = u_2^{N'+1}$ (no forces between the end atoms and external fictive atoms), it follows:

$$\begin{aligned} \kappa_i &= m_i\pi/2N'a', \\ m_i &= 0, 1, 2, ..., (N'-1). \end{aligned} \tag{3.34}$$

Next we calculate the gravitational radiation emission of the diatomic free-vibrating finite linear chain using (3.29) for the displacements out of the equilibrium position. For this we must use again the quadrupole approximation (1.5); otherwise the angular distribution of the radiation does not coincide with that of the vibrator. This means that only the low frequency modes of the chain, which also are piezoelectrically excited only, are usable for simulation of the vibrator behaviour.

The mass distribution for the chain vibrating in the m_i-th mode is given by

$$\rho(\mathbf{x}', t') = \sum_{s=1}^{N'}\{M_2\delta(\mathbf{x}' - [(2s-1)a' + u_{2m_i}^s(t')]\mathbf{e}_z) + M_1\delta(\mathbf{x}' - [2sa' + u_{1m_i}^s(t')\mathbf{e}_z)\}. \tag{3.35}$$

Assuming that the displacements out of the equilibrium position are small, this means $|u_{2m_i}^s| \ll 2sa'$ and $|u_{1m_i}^s| \ll 2sa'$, we obtain for the radiation field (1.5) after TT-projection according to (1.7)

$$h_{jk}^{(m_i)TT} = \frac{(1-n_z^2)}{r}\left\{2a'M_2\sum_{s=1}^{N'}(2s-1)\ddot{u}_{2m_i}^s + 2a'M_1\sum_{s=1}^{N'}2s\ddot{u}_{1m_i}^s\right\}e_{jk}. \tag{3.36}$$

Inserting the time derivatives of (3.29) into (3.36) and carring out the sum we get finally for the radiation field

$$h_{jk}^{(m_i)TT}(\mathrm{x},t') = \begin{cases} 0 & \text{for } m_i \text{ even,} \\ \dfrac{B_{m_i}D_{m_i}a'\Omega_{m_i}^2}{r}(1-n_z^2)\cos[\Omega_{m_i}t']\dfrac{\cos[\Psi_1(m_i)]}{\sin^2[m_i\pi/4N']} & \\ \cdot\dfrac{\{M_2A_2\cos[m_i\pi/2N'] + M_1A_1\}}{\{\cos[m_i\pi/2N'] + 1\}}e_{jk} & \text{for } m_i \text{ odd} \end{cases} \tag{3.37}$$

with the polarization tensor e_{jk} like in (2.17).

For the radial energy flux resulting from the m_i–th vibration mode we obtain with respect to (1.6):

$$T_{GW}^{0r(m_i)} = \begin{cases} 0 & \text{for } m_i \text{ even,} \\ \dfrac{(B_{m_i}D_{m_i}a')^2}{32\pi r^2}\Omega_{m_i}^6\dfrac{\cos^2[\Psi_1(m_i)]}{\sin^4[m_i\pi/4N']} & \\ \cdot\dfrac{\{M_2A_2\cos[m_i\pi/2N'] + M_1A_1\}^2}{\{\cos[m_i\pi/2N'] + 1\}^2}(1-n_z^2)^2 & \text{for } m_i \text{ odd.} \end{cases} \tag{3.38}$$

Evidently the angular distribution is identical with that of the vibrator (2.22). For the Ω–dependence we have a Ω^6–law modified, however, by a form factor dependent on $m_i(\Omega)$. The integration over the total sphere gives the following expression for the energy loss due to gravitational waves:

$$L_{GW}^{(m_i)} = \begin{cases} 0 & \text{for } m_i \text{ even,} \\ \dfrac{(B_{m_i}D_{m_i}a')^2}{15}\Omega_{m_i}^6\dfrac{\cos^2[\Psi_1(m_i)]}{\sin^4[m_i\pi/4N']} & \\ \cdot\dfrac{\{M_2A_2\cos[m_i\pi/2N'] + M_1A_1\}^2}{\{\cos[m_i\pi/2N'] + 1\}^2} & \text{for } m_i \text{ odd.} \end{cases} \tag{3.39}$$

Because only the low frequency modes are usable for simulations of the vibrator behaviour we restrict ourselves in the following to the acoustical modes with $m \ll N'(m = m_1)$. Then we obtain from (3.39) for the odd modes:

$$L_{GW}^{(m)} = \frac{16}{15}(B_{m_i}D_{m_i}A_2)^2 \cdot [N'(M_1+M_2)/2]^2(N'a')^2(2/m\pi)^4\Omega_m^6. \tag{3.40}$$

The comparison with the vibrator formula should be performed in view of the experiment in such a way that the vibrator amplitude is identified with the amplitude between the ends of the chain. For this we find from (3.29) and (3.33) in case of the acoustical branch with $m \ll N'$:

$$|C_m| \equiv \frac{1}{2}|(u_{1m}^{s=N'} - u_{2m}^{s=1})|_{\Omega_m t+\gamma_m=0} = B_m D_m A_2. \tag{3.41}$$

Insertion into (3.40) yields:

$$L_{GW}^{(m)} = \frac{16}{15}[N'(M_1 + M_2)/2]^2 \cdot (N'a')^2(2/m\pi)^4 C_m^2 \Omega_m^6. \tag{3.42}$$

Now we compare the result (3.42) for the chain with that of the vibrator, formula (2.23). We find that the vibrator simulates exactly a diatomic chain with respect to its gravitational radiation when the vibrator amplitude A is related with the amplitude C_m of the chain in the following way:

$$A = (2/m\pi)^2 C_m \tag{3.43}$$

for equal mass and length of chain and vibrator.

4. THE LABORATORY EXPERIMENT

Finally we turn our attention to the question of proving the "needle radiation" obtained in Section 2. At first we substitute the vibrator data in the essential results of Sect. 2, formulae (2.24) and (2.26), by those of the diatomic chain representing a thin piezoelectric crystal. With (3.43) and with the relations

$$M = M_c/2, \quad b = L_c/2 \tag{4.44}$$

(M_c, L_c mass and length of the chain) we find for the needle radiation and the total radiation power of a row of N piezoelectrical thin crystals (in CGS–units)

$$T_{GW}^{0y} = \frac{G}{2\pi^5 c^5} \frac{M_c^2 L_c^2}{r^2} \frac{C_m^2}{m^4} \Omega_m^6 N^2 \tag{4.45}$$

and

$$L_{GW} = \frac{2.8G}{\pi^4 c^4} M_c^2 L_c^2 \frac{C_m^2}{m^4} \Omega_m^5 N/a\,. \tag{4.46}$$

In order to estimate the available order of magnitude of the results (4.45) and (4.46) it is useful to introduce the following experimentally well controllable quantities; for the acoustic branch the dispersion relation reads $\Omega_m/\kappa = v_s$ ($\kappa = m\pi/L_c$), so that we get:

$$\begin{aligned} &m\pi = \Omega_m L_c/v_s,\ C_m = \epsilon L_c\ (\epsilon \simeq 10^{-4}[6]), \\ &N = L/a \quad (a \ll \lambda_{GW}), \end{aligned} \tag{4.47}$$

where v_s is the (nearly frequency independent) sound–velocity of the material and L the length of the oscillator row, see Fig. 5 . Herewith we obtain from (4.45) and (4.46):

$$\begin{aligned} T_{GW}^{0y} &= \frac{G}{2\pi c r^2} M_c^2 (\frac{v_s}{c})^4 \Omega_m^2 (\frac{L}{a})^2 \epsilon^2, \\ L_{GW} &= 2,8\, G M_c^2 (\frac{v_s}{c})^4 \Omega_m \frac{L}{a^2} \epsilon^2. \end{aligned} \tag{4.48}$$

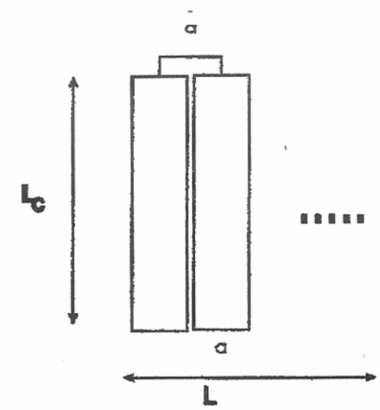

Figure 5: Oscillator row.

In the following we discuss two extreme, but perhaps possible arrangements of oscillators. As material we choose quartz with

$$\begin{aligned} &v_s = 5,76 \times 10^5 \text{cm/sec} \quad (\rho = 2,65\,\text{g/cm}^3), \\ &M_c = 10g,\ a = 10^{-3}\text{cm}. \end{aligned} \tag{4.49}$$

In the first example we take

$$\begin{aligned} &\nu = 3 \times 10^9 \text{Hz} \quad (\lambda_s = 1,9 \times 10^{-4}\text{cm}), \quad \Omega_m = 1,88 \times 10^{10}\text{Hz}, \\ &L = 10^4\text{cm}\,(N = 10^7,\ \lambda_{GW} = 10\text{cm} \gg a) \end{aligned} \tag{4.50}$$

and find from (4.48) and (4.49)

$$\begin{aligned} &L_{GW} = 4,8 \times 10^{-12}\text{erg/sec} = 4,8 \times 10^{-19}\text{watts} \\ &T^{0y}_{GW}(r = L) = 1,7 \times 10^{-17}\text{erg/sec cm}^2 = 1,7 \times 10^{-20}\text{watts/m}^2 \end{aligned} \tag{4.51}$$

within $\Delta\theta = \Delta\phi \pm 1,7^\circ$ (half-width angle).

In the second limiting case with

$$\begin{aligned} &\nu = 1,3 \times 10^{12} Hz\,(\lambda_s = 4,4 \times 10^{-7}\text{cm}),\ \Omega_m = 8,17 \times 10^{12}\text{Hz}, \\ &L = 10^4\text{cm}\,(N = 10^7, \lambda_{GW} = 2,3 \times 10^{-2}\text{cm} \gg a) \end{aligned} \tag{4.52}$$

we obtain correspondingly[2]:

$$\begin{aligned} &L_{GW} = 2,1 \times 10^{-9}\text{erg/sec} = 2,1 \times 10^{-16}\text{watts}, \\ &T^{0y}_{GW}(r = L) = 3,2 \times 10^{-12}\text{erg/sec cm}^2 = 3,2 \times 10^{-15}\text{watts/m}^2 \end{aligned} \tag{4.53}$$

within $\Delta\theta = \Delta\phi = \pm 5'$ (half-width angle). This last gravitational radiation flux (4.53) is bigger by a factor 10^3 than the electromagnetic radiation flux of

[2]For comparison: $L_{Earth} \simeq 200$ watts; $L_{Jup.} \simeq 5,3$ kwatts.

a very faint 25^{th} magnitude star ($2,4 \times 10^{-18}$ watts/m^2)! If the radiation were focussed by a High–Temperature Superconducting lens (described by another Conference paper) on a diffraction–limited, one wavelength radius circle, then the flux would be $1,3 \times 10^{-9}$ watts/m^2 in the case of 1,3 THz.

Wether these intensities are detectable perhaps as radiation–resistance of the generator or by a suitable detector possibly after focussing the radiation, must be decided by the experts. The same is true for the realization of the diameters of the piezoelectric quartz–crystals; for these it is valid:

$$M_c = L_c a d \rho \tag{4.54}$$

where d is the width of the crystals. The maximum value for d may be $d = L_c$, so that

$$L_c = \sqrt{M_c/a\rho} \simeq 61\text{cm} \tag{4.55}$$

in the case of the values (4.49); the width of 61 cm may be realized by arranging several narrow crystals side by side. But also other configurations are imaginable.

APPENDIX

We give here the determination of the half-width angles (2.25) and the integration procedure for obtaining the total radiation power (2.26) for the needle radiation of Section 2.

First we determine the half–width angle for the radiation flux over the vibrator row. Because formula (2.21) reaches noticeable values $\sim N^2$ only in a small neighbourhood of $\theta = \phi = \pi/2$, we set in (2.21) $\theta = \pi/2 + \delta$ and $\phi = \pi/2 + \epsilon$ with $\epsilon \ll 1$, $\delta \ll 1$. Then we find from (2.21):

$$\frac{\sin^2\left[N\frac{\Omega a}{2}(\sin\theta\sin\phi - 1)\right]}{\sin^2\left[\frac{\Omega a}{2}(\sin\theta\sin\phi - 1)\right]}\sin^4\theta \simeq \frac{\sin^2\left[N\frac{\Omega a}{4}(\epsilon^2+\delta^2)\right]}{\sin^2\left[\frac{\Omega a}{4}(\epsilon^2+\delta^2)\right]}. \tag{A.1}$$

With the auxiliary assumptions

$$\frac{\Omega a}{4}\epsilon^2 \ll 1 \quad \text{and} \quad \frac{\Omega a}{4}\delta^2 \ll 1 \tag{A.2}$$

it follows:

$$\frac{\sin^2\left[N\frac{\Omega a}{2}(\epsilon^2+\delta^2)\right]}{\sin^2\left[\frac{\Omega a}{2}(\epsilon^2+\delta^2)\right]} \simeq \frac{\sin^2\left[N\frac{\Omega a}{4}(\epsilon^2+\delta^2)\right]}{\left[(\Omega a/4)^2(\epsilon^2+\delta^2)^2\right]}. \tag{A.3}$$

For the half–width angle $\epsilon = \Delta\phi$, $\delta = \Delta\theta$ the last expression must be equal to $\frac{1}{2}N^2$. So we find

$$\left.\begin{array}{c}\Delta\phi\\ \Delta\theta\end{array}\right\} = \frac{2\sqrt[4]{2}A^{1/2}}{(\Omega a)^{1/2}N^{1/2}}, \tag{A.4}$$

wherein A stands for a positive constant valued in the interval $0 \leq A \leq 1$. Now we go back with (A.4) into (A.3) equated with $\frac{1}{2}N^2$ and get for A:

$$\sin\sqrt{2}A = A \Rightarrow A \simeq 1. \tag{A.5}$$

This leads to the result

$$\left.\begin{array}{l}\Delta\phi\\ \Delta\theta\end{array}\right\} = \frac{2\sqrt[4]{2}}{(\Omega a)^{1/2}N^{1/2}}. \tag{A.6}$$

Finally we prove the auxiliary conditions used above; with $\epsilon, \delta = \Delta\phi$, $\Delta\theta$ we find, using (A.6),

$$\left.\begin{array}{l}\frac{\Omega a}{4}\epsilon^2\\ \frac{\Omega a}{4}\delta^2\end{array}\right\} = \frac{\sqrt{2}}{N} \ll 1 \Leftrightarrow N \gg 1. \tag{A.7}$$

In case of a large number of vibrators in the row the auxiliary conditions are fulfilled automatically.

For the total radiation power we can write according to (2.21) under the conditions mentioned above using (A.3),

$$L_{GW} = \int T^{0r}_{GW} r^2 \sin\theta d\theta d\phi = \frac{1}{2\pi}(MbA)^2\Omega^6 \cdot \int \frac{\sin^2\left[N\frac{\Omega a}{4}(\epsilon^2+\delta^2)\right]}{(\Omega a/4)^2(\epsilon^2+\delta^2)^2} d\epsilon d\delta. \tag{A.8}$$

With the substitution $\epsilon = \hat{r}\cos\hat{\phi}, \delta = \hat{r} = \sin\hat{\phi}$ we find

$$L_{GW} = \frac{1}{2\pi}(MbA)^2\Omega^6 \int_0^{2\pi}\int_0^{\hat{r}_0} \frac{\sin^2\left[N\frac{\Omega a}{4}\hat{r}^2\right]}{(\Omega a/4)^2\hat{r}^3} d\hat{r}d\hat{\phi} = (MbA)^2\Omega^6 \int_0^{\hat{r}_0} \frac{\sin^2\left[N\frac{\Omega a}{4}\hat{r}^2\right]}{(\Omega a/4)^2\hat{r}^3} d\hat{r}, \tag{A.9}$$

wherein $\hat{r}_0$ corresponds to the half-width angles (A.6) and may be determined for simplicity in such a way that the integrand of (A.9) vanishes; thus it follows

$$\hat{r}_0 = \frac{2\sqrt{\pi}}{\sqrt{\Omega a}} N^{-1/2}. \tag{A.10}$$

Then by partial integration and the substitution $x = N(\Omega a/2)\hat{r}^2$ $(0 \leq x \leq 2\pi)$ we get

$$L_{GW} = \frac{2}{\Omega a}(MbA)^2\Omega^6 N \int_0^{2\pi} \frac{\sin x}{x} dx. \tag{A.11}$$

Because the value of the sin-integral is 1.42 [7], we obtain finally

$$L_{GW} = 2.8(MbA)^2\Omega^5 N/a. \tag{A.12}$$

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